Scheme of Work – Stage 9 Mathematics

Introduction

This document is a scheme of work created by Cambridge as a suggested plan of delivery for Cambridge Lower Secondary Mathematics stage 9. The learning objectives for the stage have been grouped into topic areas or ‘Units’.

This scheme of work assumes a term length of 10 weeks, with three terms per stage and three units per term. It has been based on the minimum length of a school year to allow flexibility. You should be able to add in more teaching time as necessary, to suit the pace of your learners and to fit the work comfortably into your own term times.

The units have been arranged in a recommended teaching order shown in the overview below. However, you are free to teach the units in any order that retains progression across the stage as your local requirements and resources dictate.

Some possible teaching and learning activities and resources are suggested for each knowledge and understanding learning objective. You should plan your lessons to include a range of activities that provide a progression of concepts and also reflect your context and the needs of your learners.

Teaching and learning in each unit should be underpinned by problem solving. For each unit, some possible activities are suggested which link the problem-solving strand to the knowledge and understanding learning objectives for the unit. To enable effective development of problem-solving skills, you should aim to include problem-solving opportunities in as many lessons as possible across the unit.

There is no obligation to follow the published Cambridge Scheme of Work in order to deliver Cambridge Lower Secondary. It has been created solely to provide an illustration of how teaching and learning might be planned across Stages 7–9. A step-by-step guide to creating your own scheme of work and implementing Cambridge Lower Secondary in your school can be found in the Cambridge Lower Secondary Teacher Guide available on the Cambridge Lower Secondary website. Blank templates are also available on the Cambridge Lower Secondary website for you to use if you wish.

Overview

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| --- | --- | --- |
| **Term 1** | **Term 2** | **Term 3** |
| Unit 1A Number, Calculation and Problem Solving | Unit 2A Number, Calculation and Problem Solving | Unit 3A Number, Measure and Problem Solving |
| Unit 1B Algebra, Geometry and Problem Solving | Unit 2B Algebra, Geometry and Problem Solving | Unit 3B Algebra, Geometry and Problem Solving |
| Unit 1C Handling Data, Measure and Problem Solving | Unit 2C Handling Data, Geometry and Problem Solving | Unit 3C Handling Data, Geometry and Problem Solving |

Unit 1A: Number, Calculation and Problem Solving

Number and Calculation

| Curriculum framework codes | Learning objectives | Activity ideas | Resources |
| --- | --- | --- | --- |
| 9Ni1 | Add, subtract, multiply and divide directed numbers. | * Learners write an operation (+, ‒, × or ÷) in each box to make the calculations correct, for example:   25 □ -15 = 40  -30 □ -25 = -55  60 □ -20 = -3  -25 □ 7 □ -5 = -13  4 □ -2 = -2  -3 □ -3 = 9  300 □ -60 = -18 000   * Use quick mental tests based on randomly generated numbers. You could ask learners to show their answers on mini whiteboards. | A means of randomly generating numbers (e.g. online random number generator, spreadsheet, number cards in a bag)  (Optional) Mini whiteboards and pens |
| 9Ni3 | Use positive, negative and zero indices and the index laws for multiplication and division of positive integer powers. | * *What is 53 × 54?Why?* Establish that the product is:   (5 × 5 × 5) × (5 × 5 × 5 × 5)  So *53 × 54 =* 57.  *What is 66 × 62?* *What do you notice?* Establish a general rule for multiplying.  Ask learners to predict a general rule for dividing. Check by simplifying 78 ÷ 72. Reinforce that the answer is 76 and not 74.   * In pairs, learners discuss what negative indices mean based on calculating divisions such as: 21 ÷ 22 and  31 ÷ 33.   Establish that:  21 ÷ 22 = 2/4 = ½, so 2-1 = 1/2  31 ÷ 33 = 3/27 = 1/9, so 3-2 = 1/9. Introduce the term ‘reciprocal’.   * Learners create number cards showing positive, negative and zero integers, e.g. 3-1, 70, 101, 23, 2-3, 3-2, 10-1. They give the cards to a partner who orders them. Learners check each other’s answers. * *What is 81 ÷ 81?* Learners discuss this in pairs.   Establish that using the division rule of indices 81 ÷ 81 = 80 and also that 81 ÷ 81 = 8 ÷ 8 = 1. Use this to establish the rule for zero powers.   * In small groups, learners find the errors in statements such as:   30 = 0  23 × 22 = 45  32 × 33 = 36  23 × 32 = 65  86 ÷ 82 = 83  *How would you explain to someone why this statement is incorrect?* |  |
| 9Np1 | Recognise the equivalence of 0.1, 1/10 and 10-1; multiply and divide whole numbers and decimals by 10 to the power of any positive or negative integer. | * *What is 3.2 multiplied by 103? …6750 ÷ 104?* Discuss answers and then ask learners to give the missing indices in examples such as:   42.5 × 10? = 425 000  21.5 ÷ 10? = 0.0215  4.6 × 10? = 46 million   * In pairs, learners use each of the numbers 101, 102, 103, 104, 105 and 106 exactly once, to make statements involving × or ÷ with six given answers (e.g. 56, 0.05, 9, 6700, 0.0034, 84 000). * In pairs, learners explore multiplications by 10 to a negative power, e.g. 740 × 10-1, 82 000 × 10-3. *What do you notice? Why is this?*   Establish that the answers are smaller than the starting values.  *What if you divide by 10 to a negative power?* Discuss the answers to example calculations, e.g. 4 ÷ 10-1 and 6 ÷ 10-2*.*  Establish that the answers are larger than the starting values because you are dividing by a fraction.   * Learners record five calculations with a given answer (e.g. 40) that involve multiplying by powers of 10. Three of the calculations should involve negative powers of 10.   They swap with a partner and mark each other’s work.  Repeat the activity for calculations involving dividing by powers of 10. |  |
| 9Nf1 | Consolidate writing a fraction in its simplest form by cancelling common factors. | * Give a false statement about simplest form, e.g.   72/90 in its simplest form is 36/45  Ask learners to comment on the statement. Establish that 36/45 can be simplified further by dividing the numerator and denominator by 9.  Establish that the most efficient way to simplify a fraction is to divide by the highest common factor (18 for 72/90), but dividing by a series of factors will lead to the same answer.   * In pairs, learners use the digits 2, 3, 4, 6 to make a variety of different fractions of the form       They write each fraction in its simplest form, expressing any improper fractions as mixed numbers. *How can you check that your mixed numbers are correct?* |  |
| 9Nf2 | Add, subtract, multiply and divide fractions, interpreting division as a multiplicative inverse, and cancelling common factors before multiplying or dividing. | * Learners find the missing digits in fraction statements such as:            * Show how to use a 4 × 5 grid divided into squares to calculate 2/5 × 3/4. (The answer is 2/5 of the yellow part.)  |  |  |  |  |  | | --- | --- | --- | --- | --- | | x | x |  |  |  | | x | x |  |  |  | | x | x |  |  |  | |  |  |  |  |  |   Establish that:    Repeat for 2/3 × 7/8, using an 8 × 3 grid.  Establish that:    Look at the workings. *What rule can we use to multiply two proper fractions?* Establish that you can multiply the numerators and denominators and then simplify if necessary.  Establish using the same two examples that calculation is easier if you cancel common factors before multiplying.   * *Dividing by 10 is the same as multiplying by which number? Dividing by 4 is the same as multiplying by which number?*   Use these examples and others to establish that dividing by a number is the same as multiplying by the reciprocal.  Work through a simple example of dividing fractions by multiplying by the reciprocal, e.g.  1/2 ÷ 1/6 = 1/2 × 6/1 = 6/2 = 3  Also use diagrams to check the answer.   |  | | --- | | 1/2 | |  |  |  |  |  | | --- | --- | --- | | 1/6 | 1/6 | 1/6 | |  |  |  |   1/2 ÷ 1/6 means how many 1/6s are there in 1/2.  Answer: 3.   * Model finding  by working out :   =  The highest common factor is 3, and dividing the numerator and denominator by 3 gives .  Show that an alternative strategy is to write both fractions with a common denominator:  =  =  Cancelling common factors gives .   * Learners find missing numbers by applying inverse operations, for example:       *Is the product bigger or smaller than the known number? What does this tell you about the missing number?*  *How can you check your answer?*   * In pairs, learners identify which of various additions, subtractions, multiplications and divisions of fractions are correct and which are incorrect, for example:         *How did you decide? Did you need to calculate?* |  |
| 9Nc1 | Extend mental methods of calculation, working with decimals, fractions, percentages and factors, using jottings where appropriate. | * Gather different mental strategies from individuals in the class and share them to create a group portfolio. * Play a loop card game with the whole class using loop cards involving mental calculations. Each learner has a card showing a calculation and an answer to a different calculation. One learner reads out their calculation. The learner with the answer on their card reads out the answer and then their calculation, and so on.   *What strategies did you use to carry out the calculation?*  Repeat to see if learners become faster. | Pre-prepared set of loop cards involving mental calculations, e.g.   |  |  | | --- | --- | | The answer is 1.75. | What is  4 ÷ 0.2? |  |  |  | | --- | --- | | The answer is 20. | What is  8 × 0.13? | |
| 9Nc2 | Solve word problems mentally. | * Display a word problem that can be solved mentally. Ask learners to discuss it in pairs. As a class identify and highlight important words and data in the problem. Share different methods and calculation strategies. * In pairs, learners discuss how to solve word problems involving repeated calculations, e.g. * What month will it be in 1000 months? * If I was facing south and turned through 765° clockwise, what direction would I be facing?   As learners are working, ask: *What strategy did you use?*  Learners make up similar problems for their partner to solve. | Pens for highlighting |

Problem Solving

Below are some possible activities linking the problem-solving strand to the knowledge and understanding learning objectives for the unit. To enable effective development of problem-solving skills, you should aim to include problem-solving opportunities in as many lessons as possible across the unit.

| Activity ideas | Resources |
| --- | --- |
| Learners work in pairs. They take four digit cards and use them to generate two two-digit numbers. They also take two sign cards to decide whether their numbers are positive or negative.  They use their numbers to make and answer an addition, a subtraction, a multiplication and a division calculation. They can use jottings to support their calculating if needed.  *What strategy did you use? How can you check your answer?* | 1–9 digit cards  Sets of cards showing + and – |
| In small groups, learners investigate what answers can be made by adding together two mixed numbers formed by arranging the six digits 2, 3, 4, 5, 6 and 9, e.g.      *What is the smallest possible answer? What is the largest possible answer? How do you know?* |  |
| Learners choose three fractions to form the bottom row of a number pyramid.    They find the top number, where each number is the product of the two numbers below it.  *Can you find a number pyramid with mixed numbers on the bottom row that gives a whole number at the top?* |  |
| In pairs, learners find  (they can use diagrams to support their calculation). The pairs feed back on the strategy they used.  Repeat for .  After the pairs have reported back, discuss different strategies such as:   * converting both fractions to improper fractions with a common denominator and then subtracting * rewriting as , and then evaluating as . |  |
| In pairs, learners discuss strategies for multiplying together two mixed numbers (e.g.  by ). For example:   * using a grid and adding the separate products to get the answer:  |  |  |  | | --- | --- | --- | | × | 2 |  | | 3 | 6 |  | |  |  |  |  * changing both fractions to improper fractions and then multiplying.   Discuss the advantages and disadvantages of different methods. |  |
| Learners decide whether statements such as the following are true or false:  3 ÷ 0.2 = 15  6 × 0.5 = 0.3  200 × 0.3 = 6  0.64 + 0.8 = 0.72  1.75 ÷ 0.25 = 0.70  1.5 + 0.05 = 1.55  1.85 – 0.05 = 1.35  They share and explain their answers with a partner. *Which incorrect answers could you spot just by looking?* (e.g. 1.75 ÷ 0.25 = 0.70 has to be incorrect because dividing by a number between 0 and 1 makes a number bigger)  Learners make up true/false decimal calculations for their partner to mark. (Learners must be able to calculate the answers mentally themselves.) *How are you trying to ‘trick’ your partner?* |  |
| In small groups, learners find a counter-example to show that each of these statements are false:   * When you multiply together two mixed numbers, the answer is never a whole number. * When you divide two fractions, the answer is always less than 1. * The product of two mixed numbers is always less than the sum of the same two numbers. |  |

Work –Mathematics

Unit 1B: Algebra, Geometry and Problem Solving

Algebra and Geometry

| Curriculum framework codes | Learning objectives | Activity ideas | Resources |
| --- | --- | --- | --- |
| 9Ae1 | Know the origins of the word *algebra* and its links to the work of the Arab mathematician Al-Khwarizmi. | * Learners research Al-Khwarizmi. They record interesting facts to share with the class. | Access to the internet for research |
| 9Ae2 | Use index notation for positive integer powers; apply the index laws for multiplication and division to simple algebraic expressions. | * Recap the rules for simplifying 37 × 32 and 48 ÷ 42. Stress the need for the expressions to have the same base. Discuss how expressions such as  and  can be simplified using the same index laws as for numbers.   Show a series of questions about simplifying expressions (with coefficient 1) using the index laws, e.g.    .  Learners show their answers on mini whiteboards.   * Pairs of learners sort a set of cards showing expressions (with coefficient 1) into groups of equivalent expressions.   For example, one group of cards could be , and , another group could be  and .   * In pairs, learners discuss how to simplify expressions such as:     .     * Learners find the missing expressions in algebraic statements such as: | Mini whiteboards and pens  Sets of cards for sorting activity |
| 9Ae3 | Construct algebraic expressions. | * Display a word problem, e.g.   Pens are sold in two sizes of boxes. A small box holds *n* pens. A large box holds 10 more pens than a small box.  A customer buys *m* small boxes and 2*m* large boxes. Find an expression for the total number of pens the customer buys.  *What is the key information in this problem?* Underline the key information. *How can we write that piece of information as an expression?*  Learners work in pairs to combine the expressions discussed into an expression for the total number of pens bought*.*   * Learners build compound shapes from rectangles with algebraic dimensions, e.g.   3*b*  2*c*  5*d*  2*a*  They derive expressions for the side lengths, perimeter and/or area of their compound shapes. | Rectangles with algebraic dimensions such as: 2*a ×* 3*b,* 2*c ×* 5*d*. |
| 9Ae4 | Simplify or transform algebraic expressions by taking out single-term common factors. | * Ask learners to simplify the results found in the activities above. * Learners mark statements such as these using a tick (🗸) or a cross (🗴):   5*n* + 10 = 5(*n* + 2)  6*m* – 3 = 3(2*m* – 0)  2*m*2 + 5*m* = *m*(22 + 5)  12*d*2 – 6*d* = 6*d*(2*d* – 1)  4*y*2 + 5*y* = 4*y* (*y* + 1)  16*p*2 + 8*p* = 4(4*p*2 + 2*p*)  Discuss as a whole class why the wrong answers are wrong. *Is there a better way of expressing 16p2 + 8p? Why?*   * Learners find different ways of writing   40*a*2*b* + 24*ab*2 in the form  ( + ).  *Which way do you think would be considered the best way? Why?* |  |
| 9As1  9As2 | Generate terms of a sequence using term-to-term and position-to-term rules.  Derive an expression to describe the *n*th term of an arithmetic sequence. | * Recap on generating the terms of arithmetic sequences, by playing ‘Guess my sequence’. Secretly write down a sequence, e.g. 7, 11, 15, 19, 23.   Explain that you are thinking of a sequence with its first term between 5 and 10 and term-to-term rule ‘Add 4’. Ask learners to write down all the possibilities for the first 5 terms of your sequence. *What is the position-to-term rule?*  Give the *n*th term of your sequence: 4*n* + 3. *Which is my sequence?*  Repeat for other arithmetic sequences.   * Learners generate the first four terms of arithmetic sequences with the following *n*th term rules:   2*n* + 3  5*n* – 1  6 – 3*n*  Discuss how the term-to-term rules of the sequences relate to the coefficients in the *n*th term rules.   * Learners work in small groups with two sets of cards, showing the first four terms of arithmetic sequences and the *n*th term rules for these sequences. Learners match the sequences with their *n*th term rules.   *Can you match any of the* n*th term cards without having to generate the terms of the sequence?* | Two sets of cards for matching activity, one set showing the first four terms of arithmetic sequences and the other set showing their *n*th term rules |
| 9Gs3 | Draw 3D shapes on isometric paper. | * Model using isometric dotty paper to draw a cube.   *How many edges does a cube have? How many can you see?* Establish that hidden edges are not represented on the diagram.  Model showing a second cube joined to the front of the first cube (deleting hidden edges).    Ask for a volunteer to show what the diagram would look like if a third cube is placed on top of the right-hand cube.    Explain that the faces at the top or on one side can be shaded to help us to interpret the diagrams more easily.  Point out that isometric paper needs to be used the right way up. It is the right way up when the ‘diamond shapes run sideways’.   * Learners join six cubes together to make a shape. They then draw their shape on dotty isometric paper. Learners swap their shapes with a partner and draw the new shape. They compare their drawings. * Give each pair some drawings of 3D shapes drawn on isometric dotty paper. Ask them to build each shape using cubes. | Interlocking cubes – 6 cubes for each learner  Rulers  Dotty isometric paper  Pre-prepared drawings of 3D shapes made of cubes  Interlocking cubes |
| 9Gs4 | Analyse 3D shapes through plans and elevations. | * Learners develop plans and front and side elevations for their shapes in the activities above. * Learners work in pairs to play ‘Guess my shape’: * Learner 1 makes a shape using exactly six cubes without Learner 2 seeing it. * Learner 2 asks Learner 1 to draw either a plan, or a front or side elevation of the shape. * Learner 2 then guesses what Learner 1’s shape looks like and makes it from cubes. The learners then swap over. | Squared paper  Interlocking cubes – 6 for each learner  Squared paper |
| 9Gs5 | Identify reflection symmetry in 3D shapes. | * Review line symmetry in 2D shapes. Then introduce the idea of a plane of symmetry in a cuboid.     Build a model of a cuboid from modelling clay and demonstrate cutting the cuboid in half vertically through a plane of symmetry to show the two identical halves.  *How many planes of symmetry does this cuboid have in total?* (3) *Where are all the planes of symmetry?*  *How many planes of symmetry would a cuboid have if its cross-section was a square?* (5)   * Learners build symmetrical models. *Does your model have more than 1 plane of symmetry?* * In pairs, learners find all the planes of symmetry of a cube. They draw diagrams on dotty isometric paper to show all the planes of symmetry, e.g.     There are 9 planes of symmetry in total. | Modelling clay  Interlocking cubes  Models of cubes  or  Modelling clay for making cubes  Dotty isometric paper |
| 9Gp1 | Tessellate triangles and quadrilaterals and relate to angle sums and half-turn rotations; know which regular polygons tessellate, and explain why others will not. | * Revise the definitions of ‘tessellation’ and ‘tessellate’. Learners work in pairs with a selection of different regular shaped tiles. They explore which shapes will tessellate with others of the same shape and which will not. * Display diagrams to show that equilateral triangles and regular hexagons tessellate, but regular pentagons do not tessellate. *Why is it not possible to cover a plane with regular pentagons?* Relate to the angle sum at a point, establishing that (as each interior angle of a regular pentagon is 108°) three pentagons joined together leave a gap of 36°, which is not large enough for another pentagon to fit in.     **gap**  *What regular shapes other than equilateral triangles and squares can cover a plane without leaving gaps?* *Why is this?* Establish that hexagons can cover a plane as their interior angle (120°) is a factor of 360°.   * Learners explore covering the plane using two or more regular polygons. For example, can learners cover the plane using: * a mixture of regular octagons and squares? * a mixture of regular hexagons and equilateral triangles? * a mixture of squares and equilateral triangles? | A range of tiles of different regular shapes, some that tessellate and some that do not  Prepared diagrams to show that equilateral triangles and hexagons tessellate, but pentagons do not tessellate |
| 9Gp4 | Enlarge 2D shapes, given a centre and positive integer scale factor; identify the scale factor of an enlargement as the ratio of the lengths of any two corresponding line segments. | * Learners enlarge a range of triangles with different positive scale factors and different centres of enlargement. *What changes when a shape is enlarged?* (side lengths) *What stays the same?* (angles) * Learners work in pairs. Give them the coordinates of a shape after a given enlargement, e.g. an enlargement of scale factor 3, centre (2, 1). Learners find the coordinates of the original object. *How can you check your answer?* * In pairs, learners discuss how to describe the single transformation that maps triangle P onto triangle Q:   Model finding the centre of enlargement by connecting vertices in corresponding positions. Establish that the scale factor can be found by finding the ratio of two corresponding side lengths.   * Give learners a selection of grids showing enlargements of shapes. Ask learners to describe each enlargement fully. | Coordinate grid or squared paper Rulers  Coordinate grid or squared paper Rulers  Large grid showing the enlargement of a triangle  Ruler  Grids showing enlargements |

Problem Solving

Below are some possible activities linking the problem-solving strand to the knowledge and understanding learning objectives for the unit. To enable effective development of problem-solving skills, you should aim to include problem-solving opportunities in as many lessons as possible across the unit.

| Activity ideas | Resources |
| --- | --- |
| Establish that the diagram shows two congruent shapes on a 100 square. One of the shapes has been placed around the number 23 and the other around the number 47.    Learners work in pairs to generalise about the total of the five numbers in shapes like this in various positions (without changing the orientation of the shape).  *What is the total of the five numbers in each of the shapes? What do you notice about the total of the five numbers in all the shapes?* (It is always a multiple of 5.) *How can you prove that by writing an algebraic expression?*  The shape is moved to a different position. The total of the 5 numbers is now 315. *Where is the shape located? How do you know?*  Challenge learners to try to find patterns in the totals of the numbers in their own shapes on a 100 square. | 100 square for each learner |
| In pairs, learners consider this ‘I think of a number’ algebraic problem:  I think of a whole number.  I add 4.  I multiply the total by 5.  I subtract 8.  I subtract the number I first thought of.  They use algebra to explain why the final number will always be a multiple of 4.  Encourage learners to record the number at each stage, e.g.  *n*  *n* + 4  5(*n* + 4) = 5*n* + 20  5*n* + 20 – 8 = 5*n* + 12  5*n* + 12 – *n* = 4*n* + 12  = 4(*n* + 3) which is a multiple of 4  Learners create ‘I think of a number’ problems for a partner to solve. They use algebra to explain why their problem works. |  |
| Learners work in pairs to find out how a sequence continues when the term-to-term rule is ‘Find the mean of the two previous terms’ and the first two terms are 5 and 1.  *What do you notice? Explore what happens when the first two terms are altered.* |  |
| Give learners a selection of elevations and plans of houses to match. *What clues did you use? Why can’t this plan belong to this house?*  You could even challenge learners to imagine and sketch a front elevation of a house based on its plan. | Printouts of matching elevations and plans of houses prepared using property websites |
| In pairs, learners find a counter-example to show that each of these statements is false:   * If a 3D shape has an odd number of vertices, it has no planes of symmetry. * A 3D shape cannot have exactly two planes of symmetry. * If an object has exactly one plane of symmetry, it must be a prism. |  |
| In small groups, learners investigate these questions:   * *Can any triangle be made to cover a plane?* * *Can any quadrilateral be made to cover a plane?*   Encourage learners to test the statements by trying to form repeating patterns from different shapes. Their patterns should be made from at least 10 shapes so that it is clear how the pattern develops, for example:  Encourage learners to justify their solutions. | Square dotty paper |

Unit 1C: Handling Data, Measure and Problem Solving

Handling Data and Measure

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| --- | --- | --- | --- |
| Curriculum framework codes | Learning objectives | Activity ideas | Resources |
| 9Ma1 | Convert between metric units of area, e.g. mm2 and cm2, cm2 and m2,and volume, e.g. mm3 and cm3, cm3 and m3; know and use the relationship 1 cm3 = 1 ml. | * In small groups, learners identify the relationships between: * m2 and cm2 * cm2 and mm2 * m3 and cm3 * cm3 and mm3   *Can you draw a diagram to show your reasoning? What could you measure using each unit? How can we convert between m2 and cm2?*   * Discuss relationships between litres, millilitres and m3, cm3 and mm3, e.g.   1 cm3 = 1 ml  1 litre = 1000 ml   * Ask learners to express a series of areas, capacities and volumes in different units. They show their answers on mini whiteboards. | Mini whiteboards and pens |
| 9Ma2 | Know that land area is measured in hectares (ha), and that  1 hectare = 10 000 m2;  convert between hectares and square metres. | * Introduce the hectare as a unit of area (1 hectare = 10 000 m2). *How can we convert between hectares and m2? Give an example. What about from m2 to hectares?* * Learners order a set of cards showing area measurements in order of size. | Sets of pre-prepared cards showing areas in different units, e.g.  0.005 ha, 4.4 m2, 4900 m2, 50 000 cm2, 4300 mm2, 400 cm2 |
| 9Mt1 | Solve problems involving average speed. | * Display the formula:   ..  Discuss with learners problems such as   * I travel at an average speed of 60 km/h for 3½ hours. How far do I travel? * I travel 24 km in 30 minutes. What is my average speed in km/h? * I drive at an average speed of 80 km/h. How long does it take me to travel 240 km?   *How did you work it out?* Reinforce how to change the subject of the formula.  Learners create problems like those above for a partner to solve. (They must be able to calculate the answers themselves so they can check them.)  *What degree of accuracy is appropriate for your solutions?*   * In pairs, learners discuss and solve problems such as:   The distance from A to B is 300 km.  Basma leaves town A at 08:00. She travels at an average speed of 75 km/h.  Pardu leaves town A at 08:15. He travels at an average speed of 80 km/h.  Who gets to town B first?  Learners share their strategies with the class.  Problems could involve those based on travel graphs, e.g. the point at which two people meet, or the average speed over part of the journey. |  |
| 9Ml1 | Solve problems involving measurements in a variety of contexts. | * Use practical outdoor activities where possible. Stress the importance of recording units as part of the results. Don't be afraid to set difficult problems, e.g. *How can we measure the height of a tree?* * In pairs, learners discuss how to convert: * 3.4 hours into hours and minutes * 2 hours 18 minutes into hours.   Learners share their strategies with the class.   * Learners answer questions involving converting units of speeds, such as:   A car travels at 20 m/s.  A motorbike travels at 70 km/h.  Which is faster? | Use whatever measuring devices you have available |
| 9Dc1  9Dc2 | Suggest a question to explore using statistical methods; identify the sets of data needed, how to collect them, sample sizes and degree of accuracy.  Identify primary or secondary sources of suitable data. | * Introduce a research question, e.g.   ‘How do the heights of 14-year-old girls compare with the heights of 14-year-old boys?’ Explain that a research question is different from a hypothesis – a hypothesis gives a statement*. Can you give a hypothesis on the same subject?* (e.g. ‘14-year-old girls are taller on average than 14-year-old boys’).   * Explain that primary data is collected by a researcher (e.g. through questionnaires and interviews), but secondary data is data that someone else has already collected (e.g. census data). Both have their own problems – discuss these. * In small groups, learners discuss research questions such as: * How has the price of petrol changed over the past 10 years? * How do most people in the school travel to school? * How do the times that boys and girls in my school get up compare? * How do life expectancies for countries in Europe compare with life expectancies in African countries?   Learners consider questions such as:   * *What hypothesis could we test?* * *Would it be better to collect primary or secondary data?* * *How could we collect the data?* * *How much data should we collect?* * *What data is needed?*   Share ideas as a whole class. |  |
| In addition to any of the activities below, learners should apply their learning to testing their hypotheses above. | | | |
| 9Dc3 | Design, trial and refine data collection sheets. | * Display the question:   ‘Don’t you agree that football is by far the best sport?’  *Is this a good questionnaire question? Why not?* (The question tries to lead people to answer yes.)  *How could the question be made better?*  Learners discuss this in pairs and then share their ideas with the class.  Establish that a possible good question, would be:  What is your favourite sport? Please tick one sport.  football □ rugby □  hockey □ netball □  tennis □ other ………..  *Why is this a good question?* (It is a simple open question. It states popular options, which will provide numerical data, but it also allows people to state an alternative sport.)   * Give a hypothesis, e.g.‘Girls that play musical instruments spend more time practising than boys.’   In small groups, learners design a data collection sheet to record data relevant to the hypothesis.  Discuss ideas as a class. Establish that it is not necessary to record the exact number of minutes spent practising. Having time intervals to choose from will make answering and recording easier.   * In small groups, learners discuss the problems with this data observation sheet:  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | |  | 0–5 mins | 8–10 mins | 10–15 mins | 18–20 mins | 20 mins or more | | Boys |  |  |  |  |  | | Girls |  |  |  |  |  |   *What would improve it?*  Establish that the intervals are not suitable as some of the boundaries overlap and the intervals do not cover all possible values. They also do not all have equal class intervals. Better intervals would be 0 ≤ *x* < 5, 5 ≤ *x* < 10, etc. to allow for every possible time. |  |
| 9Dc4 | Collect and tabulate discrete and continuous data, choosing suitable, equal class intervals where appropriate. | * Recap on the differences between discrete and continuous data. Ask learners to suggest some examples of discrete data and some examples of continuous data.   Present learners with two sets of data:   * a set of test scores (out of 20), e.g.   3, 4, 5, 5, 7, 9, 9, 10, 11, 12, 13, 13, 15, 16, 17, 19, 20   * heights of growing plants in cm   6.5, 7.2, 8.9, 10.2, 10.9, 11.4, 12.5, 13.4, 14.1, 15.0, 16.8, 19.5, 19.6, 21.7, 22.8  In pairs, learners discuss the class intervals they would use for a grouped frequency table for each set of data.  *How are the two sets of data different?* (The test scores are discrete, and the heights are continuous.) *How will your class intervals reflect this?*  Discuss learners’ suggestions. (Possible class intervals for the test scores could be:  1–5, 6–10, 11–15 and 16–20.  Possible class intervals for the heights could be  5 ≤ *x* < 10, 10 ≤ *x* < 15, etc.)   * Give learners several sets of data (some should be discrete and some continuous). Not all data is suitable for class intervals so use a range of data sets.   Learners design and complete frequency tables to summarise each set of data.  *Why do you need more than two or three class intervals?* (loss of detail) *Why shouldn’t you have too many*? (too many class intervals with no or only one entry) |  |
| 9Dp1 | Calculate statistics and select those most appropriate to the problem. | * Give learners a range of data in (ungrouped) frequency tables, e.g. data relating to the number of letters a family receives in the post over a period of 25 days:  |  |  | | --- | --- | | **Number of letters** | **Frequency** | | 0 | 6 | | 1 | 9 | | 2 | 5 | | 3 | 2 | | 4 | 1 | | 5 | 0 | | 6 | 2 |   Ask them to find the values of the mean, median and mode. *Which average do you think would be most useful in each situation?* They compare their ideas with a partner.  Establish that the mean is a good measure of average for distributions that are roughly symmetrical – it is calculated using every piece of data. But for data sets containing extreme values / outliers (or for skewed distributions), the median may be more suitable. The mode gives limited information because it represents just a few pieces of data.   * Display a grouped discrete frequency table, e.g.  |  |  | | --- | --- | | **Number of marks** | **Frequency** | | 1–5 | 5 | | 6–10 | 7 | | 11–15 | 6 | | 16–20 | 2 |   In small groups, learners discuss the modal class, the median and the mean.  *What is the modal class?*  *In which interval does the median lie?*  *What problem do we have in finding the mean? How could we estimate the mean?*  Establish that the mean can be estimated by using the midpoints of each interval to represent each interval. Show how the working can be shown by extending the table, e.g.   |  |  |  |  | | --- | --- | --- | --- | | **Number of marks** | **Frequency** | **Mid-point** | **Mid-point × frequency** | | 1–5 | 5 | 3 | 15 | | 6–10 | 7 | 8 | 56 | | 11–15 | 6 | 13 | 78 | | 16–20 | 2 | 18 | 36 | | **Totals** | **20** | **-** | **185** |      * Give learners some grouped frequency tables (some for discrete data and some for continuous data) and ask them to estimate the mean for each one. Learners also identify the modal interval and the interval in which the median lies. *What do these statistics tell you about the data?* |  |

Problem Solving

Below are some possible activities linking the problem-solving strand to the knowledge and understanding learning objectives for the unit. To enable effective development of problem-solving skills, you should aim to include problem-solving opportunities in as many lessons as possible across the unit.

| Activity ideas | Resources |
| --- | --- |
| In small groups, learners discuss things to think about when collecting primary data:   * *How could we collect primary data to investigate how the heights of 14-year-old boys and girls compare?* * *Would a questionnaire or an interview be better? Why?* * *What data would we definitely need to record?* (gender and height) * *How much data would we need to collect?* * *To what degree of accuracy should we record the heights?*   Share ideas as a whole class. Establish that:   * it is usually best to collect data from a random sample of people * you need to collect enough data to make sure results are representative of all boys and girls. |  |
| Give learners one or more of the following problems to solve:   * There are 10 singers in a choir. Their mean age is 13.5 years. Another singer aged 19 years joins the choir. What is the mean age of the 11 singers? * The mean mass of six apples is 324 g. One apple is eaten and the mean mass of the remaining apples is 316 g. What was the mass of the apple that was eaten? * I have six number cards:     What is the number on the last card if:   1. the mode is 20 2. the mean is 24 3. the median is 24 4. the median is 25.   Give a possible number for the last card if:   1. the median is 23.5 2. the median is 27 3. the range is 12. |  |

Unit 2A: Number, Calculation and Problem Solving

Number and Calculation

| Curriculum framework codes | Learning objectives | Activity ideas | Resources |
| --- | --- | --- | --- |
| 9Np2 | Round numbers to a given number of decimal places or significant figures; use to give solutions to problems with an appropriate degree of accuracy. | * In pairs, learners discuss rounding integers such as: * 8538 to 1 significant figure (The 5 digit rounds up and the zeroes preserve place value.) * 404 398 to 3 significant figures (The ‘trapped’ 0 acts as a significant figure.) * 69 563 to 2 significant figures. (For this number, the answer is the same as rounding to 1 significant figure.)   Discuss learners’ observations.   * Remind learners that rounding can be useful to help estimate the answers to calculations. Learners work in pairs to use rounding to 1 significant figure to match calculations and answers. * In pairs, learners discuss rounding decimals to the same number of significant figures and decimal places, e.g. * 0.004 103 5 to 3 significant figures and to 3 decimal places * 2.056 794 to 1 significant figure and to 1 decimal place * 1.0578 to 2 significant figures and to 2 decimal places.   *What do you notice?*  *Is it always true that a decimal rounded to the same number of significant places and decimal places are different?* (no)  *Can you give a counter-example?* (0.234 is 0.23 to 2 significant figures and also 0.23 to 2 decimal places) | 30  874.8 ÷ 32.4  16 000  367.2 × 42.5  Two sets of matching cards, one set showing calculations and the other set showing their answers when numbers are rounded to 1 significant figure, for example: |
| 9Ni2 | Estimate square roots and cube roots. | * In pairs, learners discuss how they would estimate .   *What square roots do you know that you can use to help you? Between which two integers does* *come? Why? Is it closer to 8 or closer to 9?*  Repeat for .  *How can you check your answers?*   * Ask learners to explain why, e.g.,   √30 is between 5 and 6  3√100 is between 4 and 5.   * In pairs, learners match calculations with their approximate values to the nearest whole number. | Two sets of matching cards, one set showing calculations to be estimated and the other set showing their approximate values (to the nearest whole number), for example:   |  |  |  | | --- | --- | --- | |  |  | 2 | |  |  |  | |  |  | 10 | |  |  |  | |  |  | 3 | |  |  |  | |  |  | 24 | |
| 9Np3 | Use the order of operations, including brackets and powers. | * Explain that a learner has answered the following calculations incorrectly:   🗶  🗶  🗶  *What is it that the learner doesn’t understand?* Establish the following order of operations: brackets, powers/indices, division and multiplication (from left to right), addition and subtraction (from left to right). *What are the correct answers?*   * Invite learners to the front to hold up number and sign cards to make the calculation, e.g.   16 – 7 – 2 = 11  *Do we need to insert brackets to make the answer correct?* If learners think brackets are needed give them bracket cards and ask them to position themselves in the correct positions in the calculation. *Is the calculation correct now? Why?* Repeat for other calculations, e.g.  40 – 7 + 3 × 2 = 27  5 × 22 + 2 × 3 = 66  36 ÷ 9 + 8 × 2 – 5 = 15  12 + 4 – 2 × 5 – 1 = 69   * In pairs, learners decide how to find the answers to calculations with indices, e.g.          * Encourage learners to create and share their own order of operations calculations. | Large number and operation cards |
| 9Nf3 | Solve problems involving percentage changes, choosing the correct numbers to take as 100% or as a whole, including simple problems involving personal or household finance, e.g. simple interest, discount, profit, loss and tax. | There are lots of opportunities for real-life problems here, including simulation of running a household (Which bills do you need to pay? How much are these bills? What is the average wage?) and using payslips.   * In pairs, learners discuss solving a percentage increase and a percentage decrease problem: * An item costs $1600. The cost increases by 15%. What is the new cost? * A man has 200 books. He gives 9% of his books to a friend. How many books does he have left?   Discuss learners’ strategies as a class.   * Learners work in pairs to solve multi-step word problems based on percentage increases and decreases, e.g.   Staff at a restaurant get a 20% discount on food and a 15% discount on soft drinks. Ahmed took his friends to the restaurant. The food bill came to $217 before any discount, and the soft drinks bill came to $74. How much did he pay after his staff discount?  Two years ago a factory made 72 000 shirts. Last year it made 7% fewer shirts. This year it made 5% more shirts than last year. How many shirts did the factory make this year?  *How did you find the answer? Did you need to use the calculator? What mental strategies did you use?*   * Learners work in pairs with two sets of cards. For each set, they identify the percentage change which is different from the other two. They must justify their answers. * In pairs, learners work on word problems involving profit and loss, e.g. * A man buys a car for $68 000. He sells the car 6 months later. He makes a loss of 12%. How much does the man sell the car for*?* * Deepak buys 600 pens at a cost of 90 cents each. He sells 450 pens, making a 30% profit on each of these pens. He sells the remaining 150 pens at a loss of 20%. Work out how much profit Deepak makes overall. * Explain the concept of ‘simple interest’using thisproblem:   Maria takes out a loan of $17 500. The interest rate is 1.5% each year. How much money does she need to pay back after 1 year?  *What if Maria wanted to borrow the money for 2 years … 10 years?*  Ask learners to use the internet to find the rates of interest charged by three different financial lenders. They then use these rates to find out how much someone would owe after 3 years of simple interest on a given loan.  Learners compare their strategies with another pair.   * Discuss a word problem involving calculating a percentage profit/loss, e.g.   Elena buys a painting for $17 000 and sells it a year later for $22 000. Find her percentage profit.  *How can we work this out?*  Establish that calculating % profit is similar to finding other % increases.   * Give learners details of the current tax rates and average annual salary in your country. In small groups, learners calculate the tax payable on an average salary. They then investigate the amount of tax payable on an average salary in other countries. * Ask learners to create word problems based on simple interest, tax, profit and loss for other learners to solve. They record their workings and solutions for other learners to check against. | Calculators  Two sets of cards – one set showing examples of % increases and the other showing examples of % decreases. One card in each set gives a different answer from the other two, e.g.  **Set 1**   |  |  |  | | --- | --- | --- | | From 120 cm to 135 cm | | | |  | | | | From 180 cm to 202.5 cm | | | |  | | | | From 480 cm to 552 cm | | | |  | | | | **Set 2** |  |  | | From 160 ml to 56 ml | | | |  | | | | From 150 ml to 46 ml | | | |  | | | | From 380 ml to 133 ml | | |   Access to internet for learners to research financial lending rates  (or give learners three different rates that you have researched yourself)  Calculators |
| 9Nf4 | Recognise when fractions or percentages are needed to compare different quantities. | * Groups explore the information on three different food labels. Ask learners to compare them to find out answers to questions such as:   *Which has the lowest amount of fat?*  *Which has the highest amount of protein? sugar?*  Discuss how the information is presented. *Is it easy to compare the nutritional values of different items? Why / Why not?* Establish that percentages can make comparisons easier.   * In pairs, learners prepare fact cards about two countries showing numerical data, e.g. the proportion of the country covered in forest, that can be expressed as percentages or fractions and compared. | A set of food items/labels showing nutritional information, e.g.  50 g of bread contains  Fat 1.6g  Carbohydrate 22.3g  Sugar 2.2g  Protein 4.4g  Access to internet or books for research. |
| 9Nc4  9Nc5 | Multiply by decimals, understanding where to position the decimal point by considering equivalent calculations; divide by decimals by transforming to division by an integer.  Recognise the effects of multiplying and dividing by numbers between 0 and 1. | * *What is 46 × 27?* (1242).   Discuss methods and remind learners about the standard written method of multiplication. *How can we check our answer?*  *What is 4.6 × 2.7? … 0.46 × 27? ... 0.46 × 2.7? Why?*   * *How could we calculate 1.24 × 0.84?* (e.g. Calculate 124 × 84 and then adjust.) *How could you check your answer?* (e.g. comparing with an estimate)   *How could we calculate 1.24 ÷ 0.08?* (e.g. think of the division as , which, by multiplying the numerator and denominator by 100, is the same as ).  *How could you check your answer?* (e.g. by multiplying their answer by 0.08)   * Give learners this grid of numbers.  |  |  |  |  | | --- | --- | --- | --- | | 2.4 | 6.4 | 0.34 | 2.55 | | 7.5 | 1.46 | 5.6 | 72 | | 1.56 | 105.12 | 0.65 | 35.84 |   Ask them to use the numbers to form four multiplication statements. The solutions are:  2.4 × 0.65 = 1.56  72 × 1.46 = 105.12  0.34 × 7.5 = 2.55  5.6 × 6.4 = 35.84  *What strategies are you using?*   * In pairs, learners investigate which numbers can be made by dividing any of these numbers (75, 4.5, 0.18 and 0.366) by any of these numbers (0.08, 1.5 and 0.24). *Do you notice any patterns? Do you notice any rules?* (e.g. Dividing by numbers between 0 and 1 makes a number bigger.) * Learners decide which sign (× or ÷) needs to be inserted to make a correct statement, for example:   45 🞏 10-2 = 4500  270 🞏 10-1 = 27  *How did you decide?*  Use this as a whole-class activity with mini whiteboards. | Mini whiteboards and pens |
|  |
| 9Nc3 | Consolidate use of the rules of arithmetic and inverse operations to simplify calculations. | * In pairs, learners discuss strategies for solving:   5.6 × 7.2 + 4.4 × 7.2    *How can you use factors to help you?*  Share ideas as a class. 7.2 is a factor of the first calculation so it simplifies to:  7.2 × (5.6 + 4.4) = 7.2 × 10  You can write the second calculation as  and  use understanding that dividing by 12 is the inverse of multiplying by 12.   * Ask learners to simplify and work out (mentally) the answers to a variety of calculations, e.g.   38 + 19 × 18  3.5 × 1.25 × 16  23 × 21 + 212 – 84    Encourage learners to share their answers and methods with a partner. *What rules do you need to follow?*  *How can you check your answer?*   * Learners work out the number that needs to be inserted into each calculation to make it correct without writing anything, e.g.   🞏 × 3.4 = 57.8  47.5 ÷ 🞏 = 11.2 |  |

Problem Solving

Below are some possible activities linking the problem-solving strand to the knowledge and understanding learning objectives for the unit. To enable effective development of problem-solving skills, you should aim to include problem-solving opportunities in as many lessons as possible across the unit.

| Activity ideas | Resources |
| --- | --- |
| In pairs, learners discuss problems like:   * The number 0.057\*43 rounds to 0.057 to 2 significant figures. *What are the possible values of the digit marked with \*?* * A number has five digits. Each digit is different. The number rounds to 70 000 to 1 significant figure and 2 significant figures. *What is a possible value for the number? Are there other possible values?* * A whole number rounded to 1 significant figure is 600. *What is the smallest … largest possible value of the original number?* |  |
| Give learners calculations with indices, some with correct answers and some with incorrect answers. Learners decide which answers are incorrect and explain the errors/misconceptions to a partner. |  |
| Learners work in small groups to decide whether these statements are true or false.   * If you increase an amount by 20% and then you increase the new amount by 30%, the overall increase is 50%. * If you increase an amount by 10% and then decrease the new amount by 10%, there is no overall change.   They explain their answers giving examples or counter-examples. |  |
| In pairs, learners give reasons why statements must be incorrect, for example:  4.5 × 3.7 = 16.28  56.2 × 4.8 = 2691.76  7.2 × 0.463 = 3.336  7.7 × 0.96 = 7.546  *What strategy did you use?* (e.g. estimating, looking at last digit, considering the number of decimal places.) |  |

Unit 2B: Algebra, Geometry and Problem Solving

Algebra and Geometry

| Curriculum framework codes | Learning objectives | Activity ideas | Resources |
| --- | --- | --- | --- |
| 9Ae5 | Add and subtract simple algebraic fractions. | * Ask learners to suggest how they might calculate the following:       *How did you decide on your strategy?* Establish that algebraic fractions can be added and subtracted using the same strategies as for ‘numeric fractions’.   * In pairs, learners find possible values for the numerators in this addition:        * Learners simplify expressions like:          * In small groups, learners find pairs of algebraic fractions that add to make, e.g.,   .  The fractions in each pair should have different denominators. |  |
| 9Ae6 | Derive formulae and, in simple cases, change the subject; use formulae from mathematics and other subjects. | * Discuss how to find a formula for the area of this shape formed by joining a rectangle and two semicircles.      * Give learners questions of varying difficulty about changing the subject, for example:             *How can you check your answer?*   * In pairs, learners discuss a formula relating to speed and acceleration:   *v* = *u* + *at*  *How can you make* a t*he subject?*  Give learners other examples of other scientific formulae, for example,  (speed = distance / time)  (force = mass ×  acceleration)  (pressure = force / area)  (body mass index)  (final speed from initial speed,  acceleration and distance)  Ask learners to change the subject. |  |
| 9Ae7 | Substitute positive and negative numbers into expressions and formulae. | * Review the arithmetic of negative numbers before bringing it into algebra. Encourage accurate explanations of principles. * Display the equation *y* = 5*x*2 – 2*x.*   *What is the value of* y *if* x *= -3?*  Establish that 5*x*2 means 5 times the square of *x* and that subtracting -6 is equivalent to adding 6.  Discuss how the value of *y* can be found using a calculator.  Repeat with:    *What is the value of* r *if* a *= ¾,*  b *= 12,* c *= -7 and* h *= 0.2?*   * Ask learners to substitute values into formulae from mathematics and other subjects to find the value of another variable (not necessarily the subject). | Calculators |
| 9Ae11 | Understand and use inequality signs (<, >, ≤, ≥); construct and solve linear inequalities in one variable; represent the solution set on a number line. | * Learners write down all integers that satisfy inequalities such as those below. After 10 seconds ask learners to compare their results with a partner's. Explore any errors or misconceptions with the whole group.   1 < *n* < 4  0 ≤ *n* ≤ 5  -3 < *n* ≤ 1  2 > *n* ≥ -1  1 < 3*n* < 12  *n*2 ≤ 4  Ask the class to list all the values of *x* for which *x* > 3 is true. Establish that it is an infinite list so we need a better method of representing the solution set. Introduce the notation of full/open circles to represent solution sets on a number line, e.g.  *x* < 2  *x* ≥ -1   * In pairs, learners discuss solving simple one-step inequalities (with no multiplying or dividing by negative numbers required), such as:   4*x* < 16  *x* – 5 ≥ 2    They draw the solutions as number lines.  *How can you check your values for x?* *How was solving these inequalities similar to solving equations?*   * Learners try to solve inequalities such as those below, drawing their result as a number line. Learners share their answer with a partner.   3*x* > 6  2*x* + 3 < 11  5*x* – 2 ≤ 14 – 3*x* | . |
| 9As3 | Find the inverse of a linear function. | * Define *‘*inverse’ and ‘function’. * Show a function machine to represent the function f*(x*) → 4*x* – 3:     *What function does this diagram show?*  *What is the output if* x *= 0 ….1 … 2 … 3?*  Record learners’ answers in a table:   |  |  | | --- | --- | | ***x*** | **4*x –* 3** | | 0 |  | | 1 |  | | 2 |  | | 3 |  |   Explain that an inverse function undoes the operations of a function (mapping the numbers in the right-hand column back to those in the left-hand column). *What is the inverse function to f(*x*) → 4*x *– 3?* Demonstrate by drawing an inverse function machine:       * Give a series of different linear functions for learners to find the inverses for. They show their answers on mini whiteboards. | Mini whiteboards and pens |
| 9As4 | Construct tables of values and plot the graphs of linear functions, where *y* is given implicitly in terms of *x*, rearranging the equation into the form *y* = m*x* + c; know the significance of mand find the gradient of a straight line graph. | * Recap how to produce a table of values for an equation such as *y* = 2*x* + 3.   With input from learners, model plotting and drawing the graph of *y* = 2*x* + 3.  *What is the gradient of the line? Why?*   * Using graphing software, graphical calculators or pencil and paper, ask learners to plot and label graphs of the form *y* = m*x* + c choosing their own values for m and c. Ensure that they use a range of positive, negative and zero values.   Ask learners to identify the gradient of each graph.  *What do you notice?* Establish that a graph in the form *y =* m*x +* chas gradient m.   * Show learners a list of equations of linear functions. Ask questions like:   *Which line(s) has/have a negative gradient?*  *Which line is horizontal?*  *Which line is steepest?*  *Which line has gradient equal to 1?*  Establish that equations need to be transformed into the form *y* = m*x* + c in order to identify the gradient.   * Learners work in pairs. They take turns to give the equation of a graph and their partner gives the gradient. They repeat for identifying possible equations of a graph given the gradient. | Large grid of squares    Graph paper and rulers  or  Graph drawing software  or  Graphical calculators  List of equations, e.g.  *y* = 2*x* + 3 *y* = 1  *y* = 5 – 2*x y* = *x* – 3  *2y* = *x* + 1 *y* – 4 *x* = 1 |
| 9As5 | Find the approximate solutions of a simple pair of simultaneous linear equations by finding the point of intersection of their graphs. | * Learners work in pairs. On a single pair of axes, one learner plots and draws the graph of *x* + 2*y* = 9.  The other learner draws the graph of *y* = 2*x* + 2.  *Where do the graphs intersect?*   Introduce the term ‘simultaneous equations’ (a set of equations for which there are values of the variables that solve both the equations). Explain that the coordinates of intersection represent the solution to the equations.   * Give learners several straight lines plotted on the same axes. Also give them the equations of a couple more lines to plot on the same axes. Choose lines and equations that result in several intersections.   Using the graphs, learners write the solutions to pairs of simultaneous equations. *Are the solutions whole numbers? What do you estimate the solutions to be?* | Graph paper  Rulers  For each learner, a graph showing several straight lines labelled with their equations (see activity idea) |
| 9Gs1 | Calculate the interior or exterior angle of any regular polygon; prove and use the formula for the sum of the interior angles of any polygon; prove that the sum of the exterior angles of any polygon is 360°. | * Draw a triangle and a quadrilateral. Explain the concepts of an interior angle and exterior angle.   *What do the interior angles of a triangle add up to? What about of a quadrilateral? How can you convince me that the angles in a quadrilateral add up to 360°?* Establish that any quadrilateral can be divided into two triangles. Therefore, the sum of the interior angles of any quadrilateral is 2 x 180*°* = 360*°.*   * In small groups, learners investigate the sum of the interior angles of pentagons and hexagons by dividing into triangles.   Establish that a pentagon can be divided into 3 triangles so the sum of the interior angles is 540°. Establish that a hexagon can be divided into 4 triangles and so the sum of the interior angles is 720°.  Ask learners to predict the sum of the interior angles of a decagon and of a shape with 22 sides. *What is the formula for the sum of the interior angles for a polygon with* n *sides?*   * Demonstrate that the sum of the exterior angles for any polygon is a full turn or 360°. * Learners find the missing interior or exterior angle in polygons, for example  * Learners find the size of angles in diagrams involving regular polygons, for example: |  |
| 9Gs2 | Solve problems using properties of angles, of parallel and intersecting lines, and of triangles, other polygons and circles, justifying inferences and explaining reasoning with diagrams and text. | Learners need to be able to justify their answers to problems, e.g. *x* = 50° because … Encourage this in all written and verbal responses to problems.   * In pairs, learners solve problems involving forming and solving equations based on their knowledge of the sums of interior and exterior angles, for example:  * Recap the terms 'vertically opposite angles', ‘alternate angles’ and ‘corresponding angles’. In pairs, learners find a strategy for finding angle *x* in the regular pentagon below. *What strategy did you use?* *Is there a different way to find the angle?*      * Learners find angle *a* in the diagram below. They record workings to explain each step. |  |
| 9Gp5  9Gp6 | Recognise that translations, rotations and reflections preserve length and angle, and map objects on to congruent images, and that enlargements preserve angle but not length.  Know what is needed to give a precise description of a  reflection, rotation, translation or enlargement. | * Give learners a grid with an object O and various images A, B, C … relating to a range of different transformations. Also give learners the descriptions of the transformations. They match each image with its transformation. * Give learners a grid with an object O and various images A, B, C … relating to a range of different transformations. Learners identify and describe the transformations fully*.* Encourage learners to work in pairs and to share ideas and reasoning.   *What is the same about translations, reflections and rotations?* (Angles and side lengths stay the same. They all produce congruent images.) *How can we tell if the transformation is a translation or a rotation or a reflection? How is an enlargement the same … different from a translation or a rotation or a reflection?* | Grid showing an object and various images relating to different transformations  Descriptions of the transformations represented on the grid  Grid showing an object and various images |

Problem Solving

Below are some possible activities linking the problem-solving strand to the knowledge and understanding learning objectives for the unit. To enable effective development of problem-solving skills, you should aim to include problem-solving opportunities in as many lessons as possible across the unit.

| Activity ideas | Resources |
| --- | --- |
| In pairs, learners explore:   * values for *a*, *b* and *x* such that *y* = 40 when * values for *a* and *x* such that *y* = 8 when |  |
| In pairs, learners find five different linear functions that map the number 5 to 9. Encourage learners to use a range of operations, e.g.  They work out the inverse of each function. |  |
| In pairs, learners find pairs of simultaneous equations which:   * have the solution *x* = 3, *y* = 2 * have no solution. |  |

Unit 2C: Handling Data, Geometry and Problem Solving

Handling Data and Geometry

| Curriculum framework codes | Learning objectives | Activity ideas | Resources |
| --- | --- | --- | --- |
| 9Dp2 | Select, draw, and interpret diagrams and graphs, including:  Frequenc**y**   * frequency diagrams for discrete and continuous data * line graphs for time series * scatter graphs to develop understanding of correlation * back-to-back stem-and-leaf diagrams.   Frequenc**y** | * Display frequency diagrams for discrete and continuous data, e.g.     Number of people  **Number of people on a sample of buses**    **Heights of people**  *How are the diagrams similar … different?*  Establish that for grouped continuous data, there is a continuous scale with bars drawn between the class interval boundaries (no gaps). By contrast, grouped discrete data has groups labelled underneath the bars (and there are gaps between the bars).  *Why are frequency diagrams suitable to present this data?*  Read out statements like those below. Learners show on mini whiteboards whether they agree (✓) or disagree (🗶) with these statements or whether they cannot tell (?):   * Half the buses had between 15 and 19 passengers on them. * Nine buses had 20 or more passengers. * Three people had a height between 175 cm and 180 cm. * Most people were less than 160 cm tall. * Show learners time series data suitable for displaying as a line graph, e.g. life expectancy in different years.  |  |  |  |  | | --- | --- | --- | --- | |  | 2004 | 2008 | 2012 | | Males | 71.5 | 72.4 | 73.0 | | Females | 75.8 | 76.5 | 76.9 |   Demonstrate how the data can be represented as a line graph:  Life expectancy    Males  Females  Year  *Why is a line graph suitable to present this data?*  Learners work in pairs to interpret the graph.  They then share their interpretations with the class. Establish that:   * life expectancy for females is greater than life expectancy for males * life expectancy for both males and females has increased between 2004 and 2012 * life expectancy for males has increased more than life expectancy for females. * Review simple stem-and-leaf diagrams. Then show a back-to-back stem-and-leaf diagram, e.g.   **Key: 1 | 4 | 2 represents a mark of 42% in Class 1 and a mark of 41% in Class 2**  **Class 2 Class 1**    *How are the marks for Class 2 ordered differently from those for Class 1?* (The numbers decrease from left to right – so the smallest values are still nearest to the stem.)  *Why is a back-to-back stem-and leaf diagram useful to present this data?* Establish that back-to-back stem-and-leaf diagrams are a useful way to compare two sets of data.  *How do the marks in the two classes compare?* Discuss the distributions of the marks.   * Display data suitable for displaying as a scatter graph, e.g. the heights and masses of seven people:  |  |  | | --- | --- | | **Height (cm)** | **Mass (kg)** | | 152 | 55 | | 159 | 70 | | 161 | 59 | | 166 | 78 | | 171 | 64 | | 174 | 73 | | 180 | 85 |   Model how the data can be presented on a scatter graph:  Establish that the graph shows the relationship between a person’s height and weight. *How could you describe this relationship?* Establish that the graph shows that taller people tend to be heavier than shorter people.  *Why is a scatter graph useful to present this data?* Explain that scatter graphs are useful to present two related sets of data that you want to compare.   * Show learners five scatter graphs showing relationships between variables, e.g.     Explain that a scatter diagram shows correlation between two variables if there is a relationship between the variables. Establish that correlation can be positive (one variable tends to increase as the other increases) or negative (one variable decreases as the other increases). Establish that correlation is strong if the points lie very close to a straight line.  Establish that correlation does not imply causation.   * Give learners opportunities to work in groups to use and/or collect data, and choose how best to represent it as a diagram. They present their diagrams and their interpretations of them for class discussion.   Encourage discussion of the choice of diagram used, encouraging constructive feedback using the model of two good things and one ‘it would have been even better if ...’. | Pre-prepared frequency diagrams for discrete and continuous data  Mini whiteboards and pens  Pre-prepared time series data and a matching line graph  Pre-prepared back to back stem-and-leaf diagram  Pre-prepared data suitable for displaying as a scatter graph  Large sheet of squared paper    Pre-prepared scatter graphs showing different correlations |
| 9Di1  9Di3 | Interpret tables, graphs and diagrams and make inferences to support or cast doubt on initial conjectures; have a basic understanding of correlation.  Relate results and conclusions to the original question. | * Give learners some initial hypotheses and some frequency diagrams. They decide with a partner whether the diagrams support the hypotheses or not.   Pre-prepared frequency diagrams each with an accompanying hypothesis, e.g.    Number of words remembered  **Hypothesis: Most people will remember more than 18 words.**   * In small groups, learners collect data and present it as scatter graphs. Possible ideas are: * Learners collect primary data from the class (e.g. height, hand span, arm length, length of finger, number of hours of TV watched per week). Learners investigate if there is a relationship between any of the variables by choosing two and drawing a scatter graph. * Learners explore secondary data for world data for life expectancy and e.g. wealth, literacy rate, employment rate, carbon dioxide emissions per capita. Learners investigate which variables are correlated with life expectancy. * Give learners opportunities to work in groups to collect and present data to test their own research questions and hypotheses. Learners interpret their diagrams and comment on whether they support their hypotheses. Does the rest of the class agree with them? * For the activity above, ask learners to relate their results and conclusions to their original research questions. Make sure learners understand that it doesn't really matter if your original hypothesis was right or wrong, as long as you can tell if your collected data supports it or not. | Pre-prepared frequency diagrams and related hypotheses  Access to the internet  Squared paper |
| 9Di2 | Compare two or more distributions; make inferences, using the shape of the distributions and appropriate statistics. | * Show a back-to-back stem-and-leaf diagram showing the length of leaves from two plants in mm:   **Plant A Plant B**    In pairs, learners discuss statistics that can be used to compare the lengths of the leaves of the two plants.  *What do the statistics show?*  Establish that we need a measure of average/typical values (mean, median or mode) and a measure of spread (e.g. the range).   * *What is the median leaf length of each plant? What do they tell you?* * *What is the mode leaf length of each plant? What do they tell you?* * *Which is the most representative average in this context? Why?* * *Why is the median more convenient in this situation than the mean?* * *What are the values for the range of lengths for each plant? What do they tell you?* * Give each learner two sets of data suitable for displaying on a back-to-back stem-and-leaf diagram, e.g. ages of people working in two offices. Learners: * draw a back-to-back stem-and-leaf diagram * find the median and range * write a comparison of the data. * Model how to produce a frequency polygon from a frequency diagram for continuous data by joining the midpoints of each bar.   e.g.    Show how frequency polygons are useful for comparing two distributions, e.g.  **Frequency polygons showing maximum temperature in each month in two towns**    Town B  Town A  Temperature oC  *How do the temperatures in the two towns compare?* Encourage learners to comment on which town is warmer on average and to compare the spread.   * Give learners two sets of grouped continuous data. They draw and interpret a single frequency polygon to illustrate one set of data. They then draw a frequency polygon for the second set of data and compare the two sets of data. * Learners work in small groups. Share the hypothesis:   ‘Young adults are taller than older adults’.  Give each group three (or more) frequency diagrams showing heights of people in different age groups.  Learners:   * compare the diagrams * find an estimate of the mean for each distribution * discuss whether the hypothesis seems to be correct or not.   Groups share their conclusions with the class. | Large back-to-back stem-and-leaf diagram  Two sets of data for comparison  A large frequency diagram for continuous data  Two sets of grouped continuous data  Graph paper or squared paper  Rulers |
| 9Gp3 | Transform 2D shapes by combinations of rotations, reflections and translations; describe the transformation that maps an object onto its image. | The important word here is ‘combination’. Sometimes the order matters, sometimes not.  Ensure learners understand the notation A' as the image of A, etc.   * In pairs, learners investigate what single transformation is equivalent to combinations of transformations, e.g. * a reflection in the line *x* = 3 followed by a reflection in the line *y* = 2 * a rotation through 180°, centre (0, 0), followed by a reflection in the *y*-axis * a reflection in the line *y = x* followed by a reflection in the *x*-axis * a rotation by 90° anticlockwise, centre (0, 0), followed by a translation of two squares across.   Learners need to decide on a suitable object to apply the transformations to. | Coordinate grids or squared paper |
| 9Gp2 | Use the coordinate grid to solve problems involving translations, rotations, reflections and enlargements. | * In small groups, learners investigate the following combinations of transformations in pairs. * A reflection in the line *x = a* followed by a reflection in the line *x = b* * A reflection in the line *x = a* followed by a reflection in the line *y = b*   *Are there any general statements that you can make?*   * In pairs, learners investigate two transformations: * enlargement scale factor 2, centre (0,0) * translation of 2 squares across and 1 up   Explain that the two transformations are applied to a shape one after the other. Learners investigate whether the order of the transformations makes a difference.  *Can you find a single transformation that corresponds to that order?* | Coordinate grids or squared paper  Rulers  Coordinate grids or squared paper  Rulers |

Problem Solving

Below are some possible activities linking the problem-solving strand to the knowledge and understanding learning objectives for the unit. To enable effective development of problem-solving skills, you should aim to include problem-solving opportunities in as many lessons as possible across the unit.

| Activity ideas | Resources |
| --- | --- |
| Learners create a possible back-to-back stem-and-leaf diagram to match given statistics, e.g.  Test results for two classes (marked out of 50):   |  |  | | --- | --- | | **Class 1**  Median = 26  Range = 36  Number in class = 20 | **Class 2**  Median = 31  Range = 25  Number in class = 21 |   Learners compare their diagrams. *Are they the same? Why? Why not?* |  |
| In pairs, learners discuss given pairs of variables and whether there is likely to be a positive correlation or a negative correlation or no correlation between them.  Examples could include:   * marks gained by 8 learners in two maths tests * marks in a test and the distance the learner lives from the school * marks in a test and the number of days absent from school in the past year.   Learners explain to another pair their reasoning for each answer. *Do you think there would be a strong or a weak correlation? Why?* |  |

Unit 3A: Number, Measure and Problem Solving

Number and Measure

|  |  |  |  |
| --- | --- | --- | --- |
| Curriculum framework codes | Learning objectives | Activity ideas | Resources |
| 9As6 | Use systematic trial and improvement methods to find approximate solutions of equations such as *x*2 + 2*x* = 20 (1, 2 and 7). | * In pairs, learners use trial and improvement to find the positive solution to the equation:   *x*2 + 2*x* – 4 = 66  giving the value of *x* to 1 decimal place.  *How will you record your trials systematically? What would be a good starting value to try?*  Discuss learners' strategies and model the process shown in this table:   |  |  |  | | --- | --- | --- | | ***x*** | ***x*2 + 2*x* – 4 =** | **Comment** | | 8 | 76 | Too big | | 7 | 59 | Too small | | 7.5 | 67.25 | Too big | | 7.4 | 65.56 | Too small | | 7.45 | 66.4025 | Too big |   Establish that trying a whole number first is best and, as 82 = 64, a starting value of *x* = 8 would be a good choice.  The solution is required to 1 decimal place, so the solution must be either *x* = 7.4 or *x* = 7.5. To decide between these values, try the value half way between: *x* = 7.45. This gives a value that is too big, so the solution must be less than 7.45. Establish that the solution to 1 decimal place must be *x* = 7.4.   * Give learners simple quadratic equations to solve to 1 decimal place using trial and improvement, e.g.   *x*2 + *x* + 6 = 31  *x*2 + 3*x* = 80  *x*2 – *x +* 11 = 90  *What will you use as your starting value? Why?*   * Learners use trial and improvement to find approximate solutions to problems such as: * What are the dimensions of this rectangle, which has area 30 cm2?      * The product of two consecutive numbers is 12 656. What are the two numbers? |  |
| 9Nf5 | Compare two ratios; interpret and use ratio in a range of contexts. | * In pairs, learners discuss how to solve ratio problems such as: * The ratios of men to women in two choirs are:   Choir 1 3 : 5  Choir 2 11 : 19  *Which choir has the higher proportion of men? How did you decide?*   * In a competition, all learners in Grades 8 and 9 are awarded a bronze, silver or gold certificate in the ratios:   Year 8 12 : 5 : 3  Year 9 11 : 10 : 4  *Which grade had the highest proportion of gold certificate winners? How did you decide?*  Discuss various approaches, including for the choir problem:   * Writing both ratios in unitary form (that is 1 : 1.66… and 1 : 1.727… or as 0.6 : 1 and 0.58 : 1) * Comparing the fraction of each choir that is male (3/8 and 11/30), by converting to percentages. * Give learners ratios relating to the composition of different copper alloys. Learners sort the ratios according to the proportion of copper in each.  |  |  |  |  | | --- | --- | --- | --- | | **copper** | | **:** | **zinc** | | 9 | : | 1 | | 7 | : | 3 | | 19 | : | 1 | | 4 | : | 1 | | 13 | : | 7 | | 3 | : | 2 | | 3 | : | 1 | | 2 | : | 1 |   *How did you decide on your order?*   * Model how to solve a problem involving mixing colours by scaling up the ratios, e.g.   Using the RGB (Red–Green–Blue) colour model for displaying colours on screen, the ratios of red, green and blue for a particular colour are:   * red : green = 3 : 4 * green : blue = 5 : 2   *What is the ratio of red : blue?*  Establish that you can scale up each ratio to give equal parts of green:   * red : green = 3 : 4 = 15 : 20 * green : blue = 5 : 2 = 20 : 8   So red : blue = 15 : 8 |  |
| 9Nf6 | Recognise when two quantities are directly proportional; solve problems involving proportionality, e.g. converting between different currencies. | * Learners use current exchange rates for different currencies, e.g.   1 euro = 75.7 Indian rupee  1 euro = 0.88 British pounds  1 British pound = 1.31 US dollar  They answer questions that involve a straightforward conversion between two directly linked currencies (e.g. US dollars to British pounds).  You could also include questions that involve two-stage conversions (e.g. rupees to British pounds).   * Learners solve word problems involving currency conversion, such as:   A laptop costs 320 euros in France. The same laptop costs 129 rial in Oman.  In which country is the laptop cheaper?   * In pairs, learners discuss how a biscuit recipe for 20 biscuits can be adjusted to make 25 biscuits.   To make 20 biscuits  140 g butter  150 g sugar  1 egg  40 g almonds  60 g sultanas  200 g flour  2 teaspoons mixed spice |  |
| 9Mt2 | Use compound measures to make comparisons in real-life contexts, e.g. travel graphs and value for money. | * In pairs, learners discuss which of these boxes of washing tablets offers the best value for money, e.g.   Box 1: 25 tablets for $6.35  Box 2: 40 tablets for $9.80  Discuss different approaches for comparing the costs, such as   * finding the cost of one tablet in each box * finding the cost for 5 tablets in each box * scaling up the cost of Box 1 to get the equivalent cost of 40 tablets. * Ask learners to work out which offers the best value of money in different cases, for example: * 150 ml of yoghurt for $0.94   or  0.5 l of yoghurt for $3.30   * 200 g of cheese for $1.38   or  0.45 kg of cheese for $2.84  *What strategy are you using?*  You could add extra challenge by including percentage increases in the comparison statements, e.g.  A pot of yoghurt that usually holds 0.5 l but has 25% extra free today for $3.30   * Show a travel graph, such as:     Time  Distance from home (km)  *How much faster was the journey back towards home than the journey away from home?* Discuss how the formula:    relates to the gradient of a distance-time graph.   * Give learners a travel graph of a journey. Ask them to work with a partner to describe the journey. They should include speeds, for example:   Between 1 pm and 2.30 pm, the car was travelling at an average speed of 50 km/h.  Between 2.30 pm and 2.45 pm, the car was at rest …  *Do you need to include distance as well as speed in your descriptions?* (Giving speed and time allows someone to work out the distance if they need it.)  Learners do the reverse activity to the one above. Give them a description of a journey and ask them to draw a travel graph that matches the description. |  |
| 9Ma3 | Solve problems involving the circumference and area of circles, including by using the π key of a calculator. | * In pairs, learners discuss questions that involve manipulating the formulae for circumference and area of a circle, e.g. * A circle has circumference 46 cm. What is the radius of the circle? What is its diameter? * A circle has area 415 cm2. What is the radius?   *Can you draw flow diagrams to explain the steps?*   * Give each pair of learners a set of cards showing shapes labelled with their dimensions. Learners sort the shapes into groups as follows:   Group 1: Areas that round to 100 cm2 to 1 significant figure  Group 2: Areas that round to 200 cm2 to 1 significant figure  Group 3: Areas that round to 300 cm2 to 1 significant figure  *How can you use estimates to help you to calculate?*   * With input from learners, model how to find the shaded area in this diagram. Explain that O is the midpoint of the base of the semicircle. *What do we know?* (diameter of semicircle is 12 cm; diameter of each small semi-circular cut-out part is 6 cm) *What can we find out? How?*      * Learners then find the areas and perimeters of shapes formed from circles, e.g.        * Give learners a few minutes to discuss the following word problem in pairs:   The diameter of a wheel on a bicycle is 60 cm. How many times does the wheel rotate when the bicycle travels 4 km?  With input from learners, model solving the problem.  Set learners other word problems involving measurements and circles, for example:   * Helena designs a circular lawn with a diameter of 12 m. Turf costs $5 per square metre. What is the cost of the turf she needs? * Amit’s running track is shown in the diagram.     Amit runs around the track 7 times. Show that he runs just over 2 km.   * Ghania makes circular tablecloths with a radius of 75 cm. She sews lace around the edge of each tablecloth. She has 45 m of lace. How many tablecloths can she complete? | Calculators with π key  Sets of cards showing shapes labelled with their dimensions. The areas of the shapes should fit into the groups listed on the left. The shapes described could include:   * A circle with radius 7 cm * A circle with diameter 13 cm * A circle with circumference 65 cm * A semicircle with diameter 18 cm   You could also include shapes that are not circles, e.g.   * A triangle with base 16 cm and height 12.4 cm * A square of side length 16.7 cm   Calculators with π key  Calculators with π key |
| 9Ma4 | Calculate lengths, surface areas and volumes in right-angled prisms and cylinders. | * In pairs, learners discuss how to find the volume of prisms, such as:     Discuss strategies which may include dividing the compound shapes into cuboids.  Show that the volumes can be found by finding the area of the cross-section and then multiplying by the length of the prism:  volume of prism = area of cross section xlength   * In pairs, learners discuss applying the formula for the volume of a prism to find the volumes of a range of prisms, for example:      * Give learners some diagrams of prisms, where each has a volume of 200 cm3. Ask them to find the missing side length in each diagram, for example:     square cross section   * Learners explore the following problem:   Ana makes paperweights in the shape of triangular prisms.    She paints each paperweight with two coats of paint. She has enough paint to cover an area of 6000 cm2.  How many paperweights can she paint completely?   * In small groups, learners discuss how the volume and a surface area of a cylinder could be calculated, for example:     Discuss as a class, making links with the volumes and surface areas of prisms.  Establish the formula for the volume (*V*) of a cylinder:  *V* = area of cross section *x* height  So, V = πr2h  Establish that the 'unrolled' curved surface of a cylinder is a rectangle with width equal to the circumference of the cylinder's cross-section.  So the formula for the surface area (S) of a cylinder is:  S = 2πr2 + 2πrh   * Learners solve problems such as: * A cylinder has a volume of 640 cm3. It has a radius of 6.5 cm. What is the height of the cylinder? * The total surface area of a cylinder is 640 cm2. The radius of the cylinder is 7 cm. What is the height of the cylinder? * Ask learners to investigate the maximum volume for a prism or cylinder with a given surface area and vice versa. |  |

Problem Solving

Below are some possible activities linking the problem-solving strand to the knowledge and understanding learning objectives for the unit. To enable effective development of problem-solving skills, you should aim to include problem-solving opportunities in as many lessons as possible across the unit.

| Activity ideas | Resources |
| --- | --- |
| In pairs, learners discuss strategies for solving the problem:  The ratio of men to women in a room is 4 : 3. There are 35 people in the room altogether. One more woman arrives. Find the ratio of men to women now.  Counters or other resources should be available for learners to use if they would like to. | Counters (or similar) |
| In pairs, learners discuss a problem based on a recipe, e.g.  To make 20 biscuits  140 g butter  150 g sugar  1 egg  40 g almonds  60 g sultanas  200 g flour  2 teaspoons mixed spice  What is the biggest number of biscuits that you can make if I have 3 eggs, 500 g flour and 200 g sultanas (but plenty of the other ingredients)?  *Which ingredient determines the maximum number of biscuits you can make?* (The 500 g of flour means that you can only make 2 batches of the recipe despite having enough of the other ingredients for more.)  Learners create similar problems their partner to solve. | (Optional) A variety of different biscuit recipes for learners to use for their own problem |
| Give learners a problem involving surface area to solve, such as:  The cube and the cuboid have the same volume. What is the difference in their surface areas? |  |
| Discuss the following problem with the class:  Andrei has this container:    The empty container has a mass of 150 g. 1 litre of water has a mass of 1 kg.  What is the mass of the container when it is full of water?  *What do we need to find out before we can calculate the answer?* (The volume of the container.) *How can we find that? What is the relationship between cm3 and litres?* |  |
| In pairs, learners design shapes with areas that round to 200 cm2 to 1 significant figure formed from circles. *What strategies are you using?* | Calculators with π key |
| Set problems that involve reviewing formulae for areas of other shapes as well as circles, e.g.   * A circle and a square have the same area. The square has sides of length 8 cm. Find the radius of the circle. * A circle has radius 5.5 cm. A larger circle has twice the area of the smaller one. Find the radius of the larger circle. * A circle has diameter 8 cm. The perimeter of a rectangle is approximately the same as the circumference of the circle. If the rectangle has length 8 cm, find the width of the rectangle. | Calculators with π key |
| Learners work in pairs to design a school running track that meets the following requirements:   * one lap of the track must measure 500 m * the track must have semi-circular ends. | Calculators with π key |
| In pairs, learners discuss and solve the following problem:  One litre of water is poured into each of these cylindrical containers. What is the depth of water in each cylinder? | Calculators with π key |

Unit 3B: Algebra, Geometry and Problem Solving

Algebra and Geometry

| Curriculum framework codes | Learning objectives | Activity ideas | Resources |
| --- | --- | --- | --- |
| 9Ae8 | Construct and solve linear equations with integer coefficients (with and without brackets, negative signs anywhere in the equation, positive or negative solution); solve a number problem by constructing and solving a linear equation. | * Ask learners to work in pairs to solve the equation 5*x* – 2 = 3*x* – 15.   They check their answer by creating a table of *x* and *y* values and plotting a graph.  Invite a learner to model solving the equation. Emphasise:   * the need to keep the equation balanced by applying the same operation to both sides * that it is beneficial to show each stage of the working * that it is beneficial to check that their solution satisfies the original equation. * Display the steps for solving:   20 – 4*n* = 24 – *n*  Learners add explanations of the workings after \* and show a calculation to check the answer. (Possible learner answers are shown here in red)  -4*n* = 24 – *n* – 20 \*(subtracting 20 from each side)  -4*n* = 4 – *n*  -4*n* + *n* = 4 \*(adding n to each side)  -3*n* = 4  *n* = -4/3 \*(dividing each side by -3)  20 – 4 (-4/3) = 24 – (-4/3)  20 + 16/3 = 24 + 1 + 1/3  20 +51/3 = 251/3   * Learners work in pairs to solve equations that involve expanding brackets on one or both sides, e.g.   4(2*x* – 3) + 3(*x* + 5) = 3(3*x* + 7)  5(2*x* – 1) – 2(*x* – 6) = 2*x* – 5  Learners take turns to give their partner instructions to solve an equation which their partner must follow literally.   * Learners work in pairs to solve word problems by forming and solving equations, for example: * James and Alex both think of the same number. James adds 4 to his number and then multiplies the result by 6. Alex subtracts his number from 20 and then multiplies the result by 3.   They end up with the same answer.  What number did they each start with?   * Elena has $144. Zara has $112.   Movie tickets cost $*t* each.  Elena buys 12 tickets for her family and friends, and Zara buys 8 tickets. They then each have the same amount of money left.  Find the cost of a movie ticket.   * Learners solve geometry problems by forming and solving equations, for example * This pentagon has one line of symmetry. Find the perimeter of the pentagon.      * Find the size of the smallest angle:      * This triangle and rectangle have the same area. Find the perimeter of the rectangle. | Graph paper  Rulers |
| 9Ae9 | Solve a simple pair of simultaneous linear equations by eliminating one variable. | * Model how to find values of *x* and *y* that satisfy pairs of equations such as:   3*x* + 2*y* = 13  3*x* + 6*y* = 21  Establish that, for these equations, a variable can be eliminated by subtracting the equations to find *y*. Then *y* can be substituted into one of the equations to find *x*.  Demonstrate how solutions can be checked by substituting the values into the original equations.   * Learners solve simultaneous equations by adding or subtracting the equations (no multiplying), e.g.   2*x* – *y* = 11  4*x* – *y* = 21  2*x* + 3*y* = 27  4*x* – 3*y* = 9  Ensure that learners understand that they need to decide whether adding or subtracting the equations will eliminate terms, by asking, e.g. *Would subtracting these equations help? What could you do instead?*  *How can you check your answers?* Learners could use graphing software or graphical calculators to confirm their answers.   * In pairs, learners discuss how to solve pairs of equations such as:   2*x* + *y* = 17  *x* + 5*y* = 22  2*x* – 3*y* = 5  3*x* + 5*y* = 17  Establish that equations such as these can be solved by multiplying one or both equations by a constant before adding/subtracting. With input from learners, model the process. |  |
| 9Ae10 | Expand the product of two linear expressions of the form *x* ± *n* and simplify the corresponding quadratic expression. | * Display a rectangle with dimensions (*x* + 5) cm by (*x* + 3) cm. In small groups, learners discuss how to find an expression for the area.   After time for discussion, establish that the rectangle can be divided into a square and 3 rectangles:    This means an expression for the area (in cm2) would be:  *x*2 + 5*x* + 3*x* + 15  This simplifies to *x*2 + 8*x* + 15.   * Discuss expanding the brackets in   (*x* + 4)(*x* + 2).  First consider a grid approach. Then model how the four terms can be found by multiplying each term in the first bracket by each term in the second bracket:    Give learners some double brackets to expand. These should cover a range of situations, for example:  (*x* + 3)(*x* + 7)  (*x* + 9)(*x* – 3)  (*x* + 4)(*x* – 4)  (*x* – 6)(*x* – 2)  (*x* + 5)2   * Learners find the missing values in expansions such as:   (*x* + 3)(*x* + 7) = *x*2 + 🞏*x* + 🞏  (*x* + 6)(*x* + 🞏) = *x*2 + 🞏*x* + 18  (*x* – 1)(*x* – 3) = *x*2 – 🞏*x* + 🞏  (*x* + 2)(*x* - 🞏) = *x*2 – 3*x* – 🞏  (*x* + 🞏)(*x* + 🞏) = *x*2 + 7*x* + 12  (*x* – 🞏)(*x* – 🞏) = *x*2 – 8*x* + 12   * In pairs, learners try to draw a grid diagram to represent *x2 + 9x + 20* that will allow them to express the expression as the product of two brackets: (*x* + ?)(*x* + ?)*.* This will involve trial and improvement.   After time for discussion, establish that the numbers in the brackets would need to have a product of 20 and a sum of 9, and that the expression is (*x* + 4)(*x* + 5).  Tell learners that this process is called ‘factorisation’. |  |
| 9As7  9As8 | Construct functions arising from real-life problems; draw and interpret their graphs.  Use algebraic methods to solve problems involving direct proportion, relating solutions to graphs of the equations. | * Discuss this situation with learners.   A stone is dropped from the top of a cliff. Its height (*h* metres) above the ground after *t* seconds is given by the formula:  *h* = 80 – 5*t*2  *Will the graph of h against t be a straight line? Why not?*  *What shape do you think the graph will have? How could we draw the graph accurately?*  Establish that a table of values is needed:   |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | | *t* | 0 | 1 | 2 | 3 | 4 | | *h* |  |  |  |  |  |   Ask learners to draw the graph. Make sure learners draw a curve (rather than a series of straight line segments). *How would you describe the graph?*  Learners plot the graphs of other quadratic functions describing real-life situations, for example:   * The depth (*h* cm) of water in a container after  *t* minutes, given by:   *h* = 4*t*2 (*t* between 0 and 4)   * The height (*h* metres) of a ball thrown up into the air after *t* seconds, given by:   *h* = 25*t* – 5*t*2 (t between 0 and 5)  Learners compare and describe their graphs in pairs.   * Give learners situations that require them to construct functions before they plot graphs, for example:   If a taxi journey for one person costs $4 plus $4.50 per kilometre, what is the formula for the cost *c* in terms of the distance *d*?  Draw the graph.  What is the cost of a 7.5 km journey?  What is the distance travelled if the cost is $58?  *How can you check your answers?*   * Learners draw graphs to show the relationship between quantities in direct proportion. They find the equation of each line and use their equation to find the value of the variables. Examples could include: * conversion between two units of measurement, e.g. miles and kilometres, °F and °C * the amount of an ingredient needed to make a certain number of cakes * the cooking time for chickens of different masses. * Write*:*   50 metres of material cost $210.  In small groups, learners discuss how to show the relationship between length and cost graphically.  Establish that the graph is a straight line through the origin.  Learners find a formula for the cost $*C* of *L* metres of material. They use the formula to find out how much material can be bought for $357. | Graph paper  Rulers  Graph paper  Rulers |
| 9Gs6 | Use a straight edge and compasses to:  – construct the perpendicular from a point to a line and the perpendicular from a point on a line  – inscribe squares, equilateral triangles, and regular hexagons and octagons by constructing equal divisions of a circle. | * Demonstrate the steps involved to construct a perpendicular from a point to a line. You could use geometry software or an animation for this. Establish that the perpendicular line gives the shortest distance from the point to the line.   Then, demonstrate the steps involved to construct a perpendicular to a line from a point on the line.   * Learners practise both constructions in pairs so they can check each other’s method and accuracy. * Learners give a partner precise instructions to construct a perpendicular. Their partner must follow the instructions exactly. * Draw a circle. Demonstrate how to mark 6 equal divisions around the edge of the circle. You could use geometry software or an animation for this.     Demonstrate how an equilateral triangle and a regular hexagon can be inscribed within the circle by connecting the divisions on the circumference.   * Ask learners to draw a circle and its diameter. Then ask them to construct a perpendicular bisector of the diameter. *What happens when you join the points where the diameter and the perpendicular meet the circumference?* (It creates an inscribed square.) * Once learners know how to construct an inscribed square, ask: *How could you construct an inscribed regular octagon?* After learners have had time to investigate, establish how you can inscribe a regular octagon by first constructing an inscribed square and then bisecting the angles between its diagonals. | Large pair of compasses and straight edge  Or  Suitable geometry software / animation  Rulers  Pairs of compasses  Protractors  Rulers  Pairs of compasses  Large pair of compasses and straight edge  Or  Suitable geometry software / animation |
| 9Gs7 | Know and use Pythagoras’ theorem to solve two-dimensional problems involving right-angled triangles. | * In pairs, learners solve word problems, such as: A boat sails 4 km north and then 11 km east. How far is the boat then from the starting point? * In pairs, learners investigate the areas of squares drawn on the sides of different sizes of right-angled triangles. *How are you investigating? … recording systematically?* *What patterns do you notice?*   Ask learners to use their results to predict the areas of squares, for example:  Area = ?  Area = 17 cm2  Area = 65 cm2   * Ask learners to find the value of *x* in this diagram:      * Establish Pythagoras’ theorem:     *a*2 + *b*2 = *c*2  Emphasise that the theorem connects the lengths of the sides only in right-angled triangles.   * In pairs, learners find the length of the hypotenuse of right-angled triangles in simple and then more complex examples, such as:     Encourage learners to show their working in an organised way.   * Learners solve problems based on finding non-hypotenuse sides, such as: * Find the area of this triangle:      * This diagram shows three towns, P, Q and R.     Calculate the distance between towns Q and R | Calculators |

Problem Solving

Below are some possible activities linking the problem-solving strand to the knowledge and understanding learning objectives for the unit. To enable effective development of problem-solving skills, you should aim to include problem-solving opportunities in as many lessons as possible across the unit.

| Activity ideas | Resources |
| --- | --- |
| Give each learner a ‘secret’ solution, e.g. *x* = 5, *x* = -3, *x* = 0, *x* = ½ , *x* = -¼.  They make up an equation with that solution for their partner to solve. |  |
| Learners work in pairs on puzzles like the one below. Each symbol has a value and the total of the values in each row is given. Ask learners to fill in the column totals.  *How did you work out that answer?*   |  |  |  |  |  | | --- | --- | --- | --- | --- | | ■ | ■ | ■ | ♪ | **30** | | ■ | ■ | ♪ | ♪ | **24** | | ■ | ♪ | ☺ | ☺ | **22** | | □ | ♪ | ☺ | ■ | **21** | |  |  |  |  |  | |  |
| Learners solve word problems that produce simultaneous equations, e.g.  The perimeter of a rectangle is 38 cm. The difference between the length and the width is 7 cm. Find the area of the rectangle. |  |
| In small groups, learners investigate Pythagorean triples. *Can you find any right-angled triangles for which all sides have integer lengths?* |  |
| In pairs, learners find a counter-example to show this statement is false:  ‘There are no right-angled triangles where all three sides are even numbers.’ |  |

Unit 3C: Handling Data, Geometry and Problem Solving

Handling Data and Geometry

|  |  |  |  |
| --- | --- | --- | --- |
| **Curriculum framework codes** | **Learning Objective** | **Activities** | **Resources** |
| Gp7  9Gp8 | Use bearings (angles measured clockwise from the north) to solve problems involving distance and direction.  Make and use scale drawings and interpret maps. | * Give learners an A4 map of your country with airports marked on.   Explain that a pilot wants to fly a private plane between two named airports (called Airport A and Airport B in the instructions below). *Roughly what compass direction does the plane need to fly in?*  Establish that compass directions are not accurate enough for pilots to navigate with. Explain that they use special form of angles, called ‘bearings’, instead. Bearings have the following properties:   * They are measured from north. * They are measured in a clockwise direction. * They have three digits (e.g. 040° rather than just 40°).   Ask learners to draw a north line at Airport A and a straight line from there to Airport B. They find the bearing of B from A.  Then ask learners to draw a north line at Airport B and to find the bearing of A from B.   * Learners use a map of your country with airports marked on to answer questions such as: * *Which airport will a plane reach if it leaves … and flies on a bearing of …?* * *What is the bearing of … from …?* * Learners identify which statements relating to the diagram on the right are correct and which are incorrect, for example:      * The bearing of A from B is 058° * The bearing of B from A is 058° * The bearing of A from B is 238° * The bearing of A from B is 122°. * Discuss the following problem:   The bearing of B from A is 115°.  Find the bearing of A from B.  Establish the correct reverse bearing by drawing a diagram to illustrate the situation and using the properties of parallel lines.   * In pairs, learners solve problems that involve scale drawing and bearings, such as: * A boat travels on a bearing of 100° for 80 km and then on a bearing of 195° for 45 km. Show the path of the boat on a scale drawing. How far does the boat finish from its starting point?     Bridge  Scale: 1 cm represents …… m  Swamp  Tree   * Coins are buried 120 m from the bridge on a bearing of 230°. Mark on the map where the coins are buried. * Treasure is buried on a bearing of 080° from the tree and on a bearing of 345° from the swamp. Mark on the map where the coins are buried. * How far are the coins from the swamp?   NB: Fill in the blank in the scale before giving the map to learners.   * Learners create a treasure map and instructions to follow to reach the treasure, for example   Start at the Old Tree. Walk 50 m on a bearing of 110°. Then walk 20 m on a bearing of …  Learners swap their maps and instructions with a partner who identifies where the treasure is buried. | Maps of your country printed on A4 paper with airports marked on them  Rulers  Protractors  Maps of your country printed on A4 paper with airports marked on them |
| 9Gp9 | Find by reasoning the locus of a point that moves at a given distance from a fixed point, or at a given distance from a fixed straight line. | For the first two activities, learners will need plenty of space to move around, for example in a hall or outside.   * Place an object in the middle of the open space. Ask some learners to stand less than three strides from the object. Ask some other learners to stand more than three strides away from the object. Ask the remaining learners to stand exactly three strides from the object.   *How can we describe what shape all points exactly 3 strides from the object make?* Introduce the term ‘locus’ (the path traced out by a point moving according to a rule) and establish that the locus of a point three strides from the object is a circle.   * Place a rope down the middle of the open space. Ask learners to stand two strides from the rope. *What is the locus of a point exactly two strides from the rope?* * Ask learners to draw accurately some given loci, for example: * the locus of a point exactly 3 cm from a given point * the locus of a point exactly 2.5 cm from a given line   Ensure learners consider both sides of the line.   * Give each small group a scale drawing of a map with two places (A and B) marked on. Ask learners to shade a given region, e.g. the region within 50 km of place A and within 60 km of place B. Repeat with 3 places marked on a map. *How did you calculate … km on the map? How do you know that your shaded region matches the instruction?* | Open space  Object  Open space  Rope  Rulers  Pairs of compasses  Maps with scale marked  Pre-prepared instructions for identifying regions  Rulers  Pairs of compasses |
| 9Db1 | Know that the sum of probabilities of all mutually exclusive outcomes is 1 and use this when solving probability problems. | * Recap the concept of mutually exclusive outcomes. Show a spinner:     In pairs, learners list three mutually exclusive outcomes and three outcomes which are not mutually exclusive. (Getting a 1, 2, 3 or 4 on the spinner are mutually exclusive as they cannot occur at the same time. The outcomes ‘getting a red or a 1’ are not mutually exclusive as they occur at the same time.)   * In pairs, learners explore this problem:   A spinner has four sections coloured green, red, yellow and white. The probabilities of the spinner landing on some of the sections are:   |  |  |  |  |  | | --- | --- | --- | --- | --- | | Colour | green | red | yellow | white | | Probability | 0.2 | 0.1 | 0.25 |  |  * *What is the probability that the spinner lands on white? Why?* (0.45, because mutually exclusive outcomes must add to 1.) * *What is the probability that the spinner lands on green or red? … does not land on white?* * If the spinner is spun 200 times, estimate how many times it will land on red. * In pairs, learners explore the following problems: * A bag contains 12 cards numbered 1, 2, 3, … 12.  A card is chosen from the bag at random.   Some possible outcomes are:  A = the card shows an odd number  B = the card shows the number 8  C = the card shows a multiple of 4  D = the card shows a factor of 10  E = the card shows a prime number  Which pairs of outcomes are mutually exclusive?  Which pairs of outcomes are not mutually exclusive?   * A bag contains balls that are either red or green or blue. The probabilities of picking a ball of each colour are:  |  |  |  |  | | --- | --- | --- | --- | | Colour | red | green | blue | | Probability | *x* | 2*x* | *x* + 0.2 |   What is the probability that the ball is not green?   * Learners explore the following problems individually, before comparing their answers with a partner: * A bag contains cubes coloured either black, white or blue.   The probability of picking a black cube is 0.24. The probability of picking a white cube is three times the probability of picking a blue cube.  What is the probability that the cube is black or blue?   * A spinner has three coloured sections.   The probability the spinner lands on red or yellow is 0.65. The probability the spinner lands on red or green is 0.85. The probability the spinner lands on yellow or green is 0.5.  What is the probability that the spinner lands on each coloured section? |  |
| 9Db2 | Find and record all outcomes for two successive events in a sample space diagram. | * Learners create a sample space diagram for the total of two different kinds of dice. They can choose: * dice with numbers that are not consecutive, e.g. 2, 4, 6, 8, 10, 12 * dice with numbers repeated, e.g. 1, 1, 2, 2, 3, 3 * dice that have different numbers of faces.   Learners hide the face values on their possibility space diagram and challenge a partner to identify the numbers on each dice.  They ask each other questions such as: What is the probability of:   * a total of …? * a total of more/less than …? * the most likely total? * the least likely total? * an odd total? * In pairs, learners draw a sample space diagram for possible outcomes when two 1–6 dice are thrown and the difference in the scores is found.   They then discuss whether this is a fair game:  Nikola wins if the difference is 2 or more. Mihail wins if the difference is 1 or less.  *What different fair versions of the game are possible? How do you know they are fair?*   * In small groups, learners discuss whether different games are fair or not, for example: * Bag 1 contains counters numbered 1, 2, 3, 4 and 5. Bag 2 contains counters numbered 2, 3, 4 and 5.   A counter is taken from each bag and the scores are multiplied. Player A wins if the product of the scores is more than 9. Otherwise player B wins.   * Bag 1 contains counters numbered 1, 2, 2, 3 and 4. Bag 2 contains counters numbered 2, 3, 3 and 5.   A counter is taken from each bag and the score is the maximum of the two values taken. Player A wins if the score is 3. Otherwise player B wins.   * In pairs, learners discuss the following problem:   A fair spinner has five equal sections, one red, two blue and two green.  A fair die has six faces, two red, three blue and one green.  The spinner is spun and the die is thrown.  What is the probability that the spinner and the die show the same colour?  Establish that the possible outcomes can be recorded and interpreted as:   |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | --- | |  | | Spinner | | | | | | R | B | B | G | G | | Die | R | \* | \* | \* | \* | \* | | R | \* | \* | \* | \* | \* | | B | \* | \* | \* | \* | \* | | B | \* | \* | \* | \* | \* | | B | \* | \* | \* | \* | \* | | G | \* | \* | \* | \* | \* | |  |
| 9Db3 | Understand relative frequency as an estimate of probability and use this to compare outcomes of experiments in a range of contexts. | * Explain the following probability experiment: you toss two coins and you win if both coins show heads or both coins show tails. Ask learners to predict the probability of winning the game.   Toss the coins 10 times and create a table of results, e.g.   |  |  |  |  | | --- | --- | --- | --- | | **Toss** | **Result** | **Sum of wins** | **Relative frequency** | | 1 | Win | 1 | 1 | | 2 | Win | 2 | 1 | | 3 | Lose | 2 | 0.667 | |  |  |  |  | |  |  |  |  |   Explain that the relative frequency of a win is the number of wins divided by the number of tosses.  Repeat and after each set of 10 throws record the relative frequency on a line graph, for example:    Number of tosses  Relative frequency  Establish that the relative frequencies get closer and closer (converge) to the theoretical probability as the number of tosses increases.  Ask learners to try to explain why the probability is 0.5.   * In pairs, learners consider the experiment above with 3 coins (where you win if all 3 coins show heads or all three coins show tails). *What is your prediction of the probability of winning?*   Learners show the relative frequencies after every 10 tosses on a line graph.  They make a conclusion about the probability of winning (0.25) and try to explain why this probability is correct (there are 2 ways of obtaining HHH or TTT and 8 combinations in total).   * Put 10 balls of different colours into a bag (e.g. 5 red, 4 blue and 1 green).   Take a ball out of the bag and show the colour to the class. Place the ball back into the bag. Repeat until you have made 10 draws. Invite a learner to record the outcomes in a large tally chart.  *What is the relative frequency for each colour? How many balls of each colour do you think there are in the bag? Why?*  Make a further 10 draws and add the outcomes to the tally chart.  *What is the relative frequency for each colour? How many balls of each colour do you think there are in the bag now? Why?*  Repeat until a total of 40 draws have been made. Discuss the experimental probabilities for each colour of ball. Show the contents of the bag. *How do the experimental probabilities compare with the theoretical probabilities?*   * Learners consider the following probabilities of drawing different coloured balls from a bag:  |  |  |  |  | | --- | --- | --- | --- | | Colour | red | yellow | green | | Probability | 0.25 | 0.35 | 0.40 |   If there are 60 balls in the bag, how many balls are there of each colour? Based on the probabilities, what other total numbers of balls could be in the bag? (20, 40, (60), 80 …) | Coins |

Problem Solving

Below are some possible activities linking the problem-solving strand to the knowledge and understanding learning objectives for the unit. To enable effective development of problem-solving skills, you should aim to include problem-solving opportunities in as many lessons as possible across the unit.

| Activities | Resources |
| --- | --- |
| Display two labelled drawings of fields with posts marked in the centre.      In small groups, learners investigate this problem:  A goat is fixed to a post in the centre of a field by a rope 10 m long. Shade the area of the field the goat can reach in each of the two fields.  They draw scale diagrams of the fields and shade the region of the field that the goat can reach. *How did you decide what scale to use? How can you calculate the size of your shaded region?*  Learners then consider other situations where a goat is fixed by a rope to a post, e.g.   * if the post is in the corner of the field * if the post is fixed to one side of the field * if there is a shed in the field and the post is at one corner of the shed. |  |
| Give learners a treasure map and two clues to locate the treasure, for example:   * The treasure is exactly 10 m from the river. * The treasure is exactly 15 m from the tree.   Learners mark the possible positions of the treasure on the map. *How do you know that you have marked all the possible positions?* | Treasure map with scale marked |
| In small groups, learners design their own fair game involving throwing a pair of dice or drawing counters from bags. (They could also design a game that is not fair.) Learners should explain why their game is fair (or not fair).  Groups play each other's games and give feedback. They find the probability of ‘winning’ and ‘not winning’ for the game they are playing. | Dice (ideally with different numbers of sides)  A variety of different coloured counters  Bags |
| Ask learners to solve the following problem:  What is the smallest possible number of balls of each colour in the bag?   |  |  |  |  | | --- | --- | --- | --- | | Colour | yellow | black | green | | Probability | 0.136 | 0.224 | 0.64 | |  |