**Scheme of Work – Stage 7 Mathematics**

Introduction

This document is a scheme of work created by Cambridge as a suggested plan of delivery for Cambridge Lower Secondary Mathematics Stage 7. The learning objectives for the stage have been grouped into topic areas or ‘Units’.

This scheme of work assumes a term length of 10 weeks, with three terms per stage and three units per term. It has been based on the minimum length of a school year to allow flexibility. You should be able to add in more teaching time as necessary, to suit the pace of your learners and to fit the work comfortably into your own term times.

The units have been arranged in a recommended teaching order shown in the overview below. However, you are free to teach the units in any order that retains progression across the stage as your local requirements and resources dictate.

Some possible teaching and learning activities and resources are suggested for each knowledge and understanding learning objective. You should plan your lessons to include a range of activities that provide a progression of concepts and also reflect your context and the needs of your learners.

Teaching and learning in each unit should be underpinned by problem solving. For each unit, some possible activities are suggested which link the problem-solving strand to the knowledge and understanding learning objectives for the unit. To enable effective development of problem-solving skills, you should aim to include problem-solving opportunities in as many lessons as possible across the unit.

There is no obligation to follow the published Cambridge Scheme of Work in order to deliver Cambridge Lower Secondary. It has been created solely to provide an illustration of how teaching and learning might be planned across Stages 7–9. A step-by-step guide to creating your own scheme of work and implementing Cambridge Lower Secondary in your school can be found in the Cambridge Lower Secondary Teacher Guide available on the Cambridge Lower Secondary website. Blank templates are also available on the Cambridge Lower Secondary website for you to use if you wish.

Overview

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| --- | --- | --- |
| Term 1 | Term 2 | Term 3 |
| Unit 1A Number, Calculation and Problem Solving | Unit 2A Number, Calculation, Measure and Problem Solving | Unit 3A Number, Calculation and Problem Solving |
| Unit 1B Algebra, Measure and Problem Solving | Unit 2B Algebra, Measure and Problem Solving | Unit 3B Measure and Problem Solving |
| Unit 1C Handling Data, Geometry and Problem Solving | Unit 2C Handling Data, Geometry and Problem Solving | Unit 3C Handling Data, Geometry and Problem Solving |

Unit 1A: Number, Calculation and Problem Solving

Number and Calculation

| Curriculum framework codes | Learning objectives | Activity ideas | Resources |
| --- | --- | --- | --- |
| 7Nc1 | Consolidate the rapid recall of number facts, including positive integer complements to 100, multiplication facts to 10 × 10 and associated division facts. | * Use a whole-class activity to consolidate finding integer complements to 100. Say a number, e.g. 45. Learners respond rapidly with the complement to 100 (55). Then, to emphasise the connection, say 55. Learners respond with the complement. Repeat with other numbers.
* Consolidate multiplication facts to 10 × 10. In pairs, learners roll two 10-sided dice. The first learner to give the product of the two numbers scores a point if they are correct, or loses a point if they are incorrect. The winner is the first learner to score 10 points.

You could adapt this game so that learners score an extra point if they give a correct matching division fact. * Learners play a matching card game in groups of 3 or 4. They arrange their cards face down. They take turns to turn over two cards. If the cards match, the learner keeps that pair, explains how they know they are a pair and then has another turn. If the cards do not match, they are returned to their positions face down. The winner is the person who collects most pairs.
* Split the class into four teams. Ask a number fact question, e.g.

 What is 6 × 7? … 81 ÷ 9? What do we need to add to 63 to make 100? What is 100 – 89?The first team with a hand up and a correct answer scores a point for their team. Repeat several times.  | 10-sided diceSets of cards, showing number fact questions and matching answers on separate cards, e.g.

|  |  |  |
| --- | --- | --- |
| 6 × 7 |  | 42 |
| 70 + ? = 100 |  | 30 |

 |
| 7Nc7 | Use the order of operations, including brackets, to work out simple calculations. | * Give learners a mixed operation calculation to solve mentally, e.g.

 2 + 2 × 25 + 26 There may be a difference of opinion, with 78, 104 and 126 being put forward as possible answers. (If not, put forward the answers 104 and 126 for learners to comment on.)  Present the order of operations:* brackets
* indices (e.g. squares)
* multiplication and division (from left to right)
* addition and subtraction (left to right).

Ask learners to recalculate now they know the order of operations.* Explain that you are planning packed lunches for a school trip. There will be two classes – one class of 25 and one class of 26 – and one teacher. You are planning two bread rolls for each person. *How can we make sure the order of the calculation matches the problem?* (By adding brackets: 2 + 2 × (25 + 26).)
 |  |
| 7Np1 | Interpret decimal notation and place value; multiply and divide whole numbers and decimals by 10, 100 or 1000. | * Use a number line with place value headings and moveable cards with single digits on them to discuss place value. Learners investigate how the digits move in relation to the decimal point when multiplied or divided by the powers of ten.
* In small groups, learners discuss what happens when a given whole number is multiplied by 10, 100 or 1000. Then give them a decimal starting number. *How do your rules apply to decimals?*

They repeat for dividing a given whole number and a given decimal number by 10, 100 or 1000. *What happens to the digit in the tens place when it is multiplied by 100? What happens to the digit in the units place when it is divided by 10?** Learners work in pairs with two packs of cards – one showing numbers; the other showing number operations (e.g. × 10, ÷ 100). They take turns to take one of each card and state the answer. If the answer is correct, they score one point.
* Encourage learners to generalise about multiplying and dividing by 10, 100 and 1000: How would you explain to learners in stage 6 what happens when you multiply … divide any number by 10 … 100 … 1000?
 | Number cardsNumber linesTwo sets of cards per pair. One set showing ÷ 1000, ÷ 100, ÷ 10, × 1000, × 100 or × 10; the other set showing a range of whole and decimal numbers from 0 to 10 000 |
| Np2 | Order decimals including measurements, changing these to the same units. | * Give each learner a decimal card. Ask them to line up in order of the size of their decimals, smallest on the far left and biggest on the far right.
* Display four decimals for individuals to order from smallest to biggest. They explain their strategy to a partner.
* Use a conversion table to help remind learners how to convert between km, m, cm and mm. Establish that, e.g.

 7000 m = 7 km 6 m = 600 cm = 6000 mm 49 cm = 0.49 m 732 mm = 0.732 m

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| 1000 | 100 | 10 | 1 | 0.1 | 0.01 | 0.001 |
| **km** | **-** | **-** | **m** | **-** | **cm** | **mm** |
| 7 | 0 | 0 | 0 |  |  |  |
|  |  |  | 6 |  |  |  |
|  |  |  |  | 4 | 9 |  |
|  |  |  |  | 7 | 3 | 2 |

Give learners a set of four or five cards, each showing a measurement. They arrange the cards in order of size, converting the units as required. | Large cards showing decimalsSet of cards each with a measurement e.g. 3 km, 3500 m, 3.25 km, 3 km 500 m |
| 7Np3 | Round whole numbers to the nearest 10, 100 or 1000 and decimals, including measurements, to the nearest whole number or one decimal place. | * Use a number line to investigate rounding by placing, for example, a decimal on the line and deciding which whole number it is closest to. Learners devise rules for rounding.
* Use examples from the conversion table above to recap on rounding to the nearest tenth or whole unit, e.g. *What is 0.732 m to the nearest metre? … to one decimal place? Why? Which digit helps you to decide?*
* Provide a range of objects and weighing scales. Learners estimate, and then measure, the mass of the objects. They decide on the most appropriate degree of rounding to use for each object.
* Learners sort through newspaper headlines that use numbers, discussing whether these numbers are accurate, or have been rounded, and to what degree. Sometimes actual numbers are included in the newspaper article: encourage learners to look for these.
 | Number lineVariety of objects for weighingWeighing scalesNewspaper articles and headlines that use numbers  |
| 7Nc2 | Use known facts and place value to multiply and divide two-digit numbers by a single-digit number, e.g. 45 × 6, 96 ÷ 6. | * Create a class spider diagram on a large sheet of paper (or the board). In the middle write a calculation, such as 58 × 8. Ask learners to suggest ways to work out this calculation (e.g. 60 × 8 – 2 × 8) and add each valid way to the diagram. Establish that it is possible do perform a calculation that looks difficult quite easily by using known number facts.
* Ask learners to make jottings as they use known facts to multiply and divide 2-digit numbers by single-digit numbers. Discuss how they worked out their answers, e.g. by partitioning:

 45 × 6 = (40 × 6) + (5 × 6)  96 ÷ 6 = (90 ÷ 6) + (6 ÷ 6)* Make links with learners’ number fact knowledge as well as knowledge of mathematical properties, patterns and relationships of numbers.
* Ask learners to formulate their own multiplications and divisions of 2-digit numbers by single-digit numbers and calculate the answers. They swap with another learner who checks the answers using inverse operations.
 | Large sheet of paperMarker pens |
| 7Nc3 | Know and apply tests of divisibility by 2, 3, 5, 6, 8, 9, 10 and 100. | * Learners investigate and describe patterns in multiples of 3, 6 and 9 in a multiplication table, e.g. multiples of 3 have digits which add up to a multiple of 3; all multiples of 6 are also multiples of 3.
* Recap on tests of divisibility for 2, 4, 5 and 10 by asking questions such as: *Is 96 a multiple of 4? Why? Can you give a multiple of 5 that’s greater than 1000? How did you decide?*

Then extend to tests of divisibility for 3, 6, 8 and 9. Highlight multiples on a 100 square and ask, *How can you decide whether a number is exactly divisible by 3 … 6 … 8 … 9?* Ask questions as necessary to guide learners’ thinking, e.g. *What do you notice about the sum of the digits in multiples of 3 … 9?* (Their sum is a multiple of 3 … 9 or 18.) *What other multiples do the multiples of 6 … 8 have?* (They are multiples of 2 and 3 … 2 and 4.) * Give learners a number greater than 100. *Which numbers is it divisible by?* Ask learners to explain to the class how they decided on their answers.
 | Multiplication gridLarge 100 square |
| 7Nc4 | Use known facts and place value to multiply simple decimals by one-digit numbers, e.g. 0.8 × 6. | * Learners practise deriving the answers to simple decimal multiplications using known facts and place value, e.g.

 What is 0.8 × 6?  0.8 × 6 = 0.8 + 0.8 + 0.8 + 0.8 + 0.8 + 0.8  or  8 × 6 = 48, so 0.8 × 6 = 4.8 because 0.8 is ten times smaller than 8.* Ask learners to formulate their own multiplications and divisions of simple decimal numbers by single-digit numbers and calculate the answers. They swap with another learner who checks the answers using inverse operations.
 |  |
| 7Nf1 | Recognise the equivalence of simple fractions, decimals and percentages. | * In pairs, learners use paper, card and counters to make visual representations of fractions.
* Learners identify percentages as parts per 100 by colouring 10 × 10 squares. *What fraction of the whole is that? What decimal is it? Why?*
* In pairs, learners make equivalence chains using fractions, decimals and percentages, e.g.

 1/4 = 0.25 = 25%Give learners a card showing a fraction, decimal or percentage and ask them to find another two links in their chain. *How do you know that your three numbers are equivalent?** Learners play equivalence dominoes in pairs. (Each learner takes six dominoes. They take turns to put down a domino so that it joins with a domino already laid that has a matching value on the end. When learners don’t have a matching domino, they pick up a new domino from the set until they do. They winner is the first learner to lay down all of their dominoes.)
 | Paper, card, countersCards showing fractions, decimals or percentagesEquivalence dominoes |
| 7Nf2 | Simplify fractions by cancelling common factors and identify equivalent fractions; change an improper fraction to a mixed number, and vice versa; convert terminating decimals to fractions, e.g. 0.23 = 23/100. | * Use paper, card and counters. Learners cut circles (‘pies’ or ‘pizzas’) into different numbers of equal slices to make comparisons between fractions.
* Ensure that the learners understand that the denominator of the fraction is the ‘name’ of the fraction and represents the number of equal parts the whole is divided into, and that the numerator shows how many of these parts are being used.
* Learners discover equivalent fractions by further dividing each slice of the pie. Find fractions of counters and paper strips by dividing into equal parts and then selecting the required number of parts.
* Model how to cancel common factors to find the simplest form of a fraction, e.g.

 24 ~~6~~ × 2 × 2 4== 30 ~~6~~ × 5 5 *What are the factors of the numerator … denominator? What are the common factors? Why can you cancel them?*  Give learners other examples to try themselves. * Learners play a matching game where they match improper fractions to the equivalent mixed numbers. Each small group has a set of cards. They spread the cards out face down on the table. Learners take turns to turn over two cards. If the two cards match, they keep them and take another turn. If the two cards don't match, play passes to the next player. The winner is the learner who collects most cards.
 | Paper circlesPaper stripsMatching cards showing improper fractions and matching mixed numbers on separate cards, e.g. 2¼ and 9/4 |
| 7Nf3 | Compare two fractions by using diagrams, or by using a calculator to convert the fractions to decimals, e.g. 3/5 and 13/20. | * Give learners a set of 4 × 5 grids divided into squares. Ask them to shade 3/5 of one rectangle and 13/20 ofanother.

 *Which fraction is greater,3/5 or 13/20?* Establish that using the grid, you are effectively converting the fractions to fractions with the same denominator, which makes them easier to compare*.* * Learners use 4 × 5 grids to compare other fractions with denominators of 2, 4, 5, 10 and 20.
* Remind learners about the equivalence of fractions and decimals. Demonstrate the use of a calculator to convert a fraction to a decimal. To support learners’ understanding, initially use an example that learners can also check mentally, e.g. 3/5 is 3 ÷ 5, which is 0.6. We know this is correct because 3/5 = 6/10 = 0.6.

Ask learners to work in pairs. Give each pair a set of fraction cards, a number line and a calculator. Ask learners to take each fraction card in turn, convert to a decimal and place on the number line.* Learners work in groups of four. One learner shows two fraction cards. The other three learners shout out the fraction that they think is the largest (or smallest, depending on the choice made beforehand). They check using a calculator. The fastest learner gets the cards and is the next to draw two cards. The winner is the learner with the most cards.
 | 4 × 5 rectangles divided into 20 squares One set of fraction cards per pairA calculator per pairA number line per pairFraction cardsCalculators |
| 7Ni1 | Recognise negative numbers as positions on a number line, and order, add and subtract positive and negative numbers in context. | * Give each learner a different integer on a card. Include negative and positive integers. Ask them to order themselves along an imaginary number line according to a number they have been given. Discuss the relative sizes of gaps.
* Review contexts where negative numbers are found (e.g. temperature, debt). Use the large floor thermometer to establish that, for example, -6˚ is below 0˚, and +6˚ is above 0˚.

Ask questions such as: *If you start at 0˚, and it gets colder, which direction do you move on the thermometer?* *If it was -6˚ but the temperature went down by 2˚, what is the new temperature?*Model adding and subtracting positive and negative numbers, inviting learners to model calculations by moving along a floor thermometer line. First model the effect of adding and subtracting positive integers and then discuss calculations such as 2° + (-5°) and 2° – (-5°) (i.e. the difference between 2° and negative 5°) to establish that:  To add a negative number, you move to the left / down the thermometer.  To subtract a negative number, you move to the right / up the thermometer.* Display a large thermometer. Give a series of calculations, e.g.

8° – 12°, 7° + -2°, 5° – -3°. Learners show their answers on mini whiteboards. | Large cards each showing a different positive/negative numberLarge floor thermometer diagramLarge thermometer diagramMini whiteboards |

Problem Solving

Below are some possible activities linking the problem-solving strand to the knowledge and understanding learning objectives for the unit. To enable effective development of problem-solving skills, you should aim to include problem-solving opportunities in as many lessons as possible across the unit.

| **Activity ideas** | **Resources** |
| --- | --- |
| At the animal sanctuary 4/6 of the animals are cats. 1/2 of the cats are male. What fraction of the animals at the sanctuary are male cats? |  |
| Give each learner a set of 0–9 digit cards.They use their cards to make numbers the statements below. They can use each card only once.a) To the nearest 10, I round to 20. \_\_\_\_\_\_\_\_b) To the nearest 10, I round to 10. \_\_\_\_\_\_\_\_c) To the nearest 100, I round to 7200. \_\_\_\_\_\_\_\_d) To the nearest 1000, I round to 1000. \_\_\_\_\_\_\_\_ | 0–9 digit cards |
| Learners work in pairs to find a subtraction which uses all the digits 3, 4, 5, 6, 7 and has the smallest positive answer. |  |
| Learners use their knowledge of properties of numbers to find two consecutive numbers with a product of 702. They then try to establish a method that will work for any product. |  |
| Learners complete the addition □□ + □□ = □□, using each of the digits 1, 2, 3, 4, 5, and 8 only once each. |  |
| Learners see how many different ways they can make a calculator display given numbers without using specified ‘broken’ keys:* 77 without using the 7 key
* 975 without using any of the odd number keys
 | Calculators |
| A newspaper headline stated that nearly 20 thousand people visited an attraction. The actual number was 15 437. Learners discuss whether the headline was correct and their reasoning. |  |
| Learners, in pairs, investigate patterns of decimals produced by converting fractions, e.g. 1/9, 2/9, 3/9. They predict what the next decimal will be, and their partner checks by using the calculator. They try other patterns: 1/11, 2/11, 3/11……1/8, 2/8, 3/8…….1/7, 2/7, 3/7, challenging their partner to give the next decimal in the pattern. | Calculators  |

**Unit 1B: Algebra, Measure and Problem Solving**

Algebra and Measure

| Curriculum framework codes | Learning objectives | Activity ideas | Resources |
| --- | --- | --- | --- |
| 7Ml1 | Choose suitable units of measurement to estimate, measure, calculate and solve problems in everyday contexts. | * Ask*: Where might we use measurements related to length/mass/capacity? What units would you use? What other units do you know? Which is the smallest/biggest unit?*
* Provide learners with a set of everyday objects and various measuring instruments. Ask them to work in pairs to measure the objects and order them in some way. Tell them to decide for themselves what they will measure (length or mass) and how exactly they will measure them. Ask some pairs to report to the class how they compared their objects and what units they used. They also ask the class to estimate the size of one of the objects before revealing the actual measurement.
* Provide learners with two sets of cards: one showing pictures of everyday objects and the other showing units. Ask them to match a suitable unit card to each object.

Discuss the matches as a class. | A set of everyday objects (e.g. a book, a jug, a table, a glass, a pencil, a mobile phone and a ball)Measuring instruments: tape measures or rulers, measuring scalesMatching card sets: * pictures of objects (e.g. pencil, ball, book, chair, desk, sugar cube)
* unit cards (e.g. g, kg, t; or mm, cm, m and km; or l and ml)
 |
| 7Ae1 | Use letters to represent unknown numbers or variables; know the meanings of the words *term, expression* and *equation*. | * Practise using mathematical vocabulary with the whole class. Ask learners to identify terms, expressions, variables and equations from various displayed examples.
* Provide examples of terms, expressions and equations. In groups, learners take turns to turn over a card from a pile. They state whether the card shows a term, an expression or an equation.
* Reinforce the idea of an unknown by providing learners with simple equations to solve, e.g. 23 + ❑ = 79 4 × ❑ – 7 = 17.Also ask:

*If* ⬤ *+* ⏹ *= 6, what could* ⬤ *and* ⏹ *represent?* Link to the use of letters in mathematics. So: 23 + ❑ = 79 could be written as  23 + *n* = 79 ⬤ + ⏹ = 6 could be written as*a* + *b* = 6.Point out in the examples above that:* there is only one possible answer for *n,* i.e. *n* can only take one value.
* there is more than one possible answer for *a* and *b*, i.e. *a* and *b* can take many values.
* In pairs, learners take two numbers and a letter/shape. They create a simple equation using +, –, × or ÷ and =, and deduce the value of the letter/shape.
* Learners use cards to find matching simple expressions and word statements, e.g.

 ‘Add 7 to a number.’ would be matched with ‘n + 7’ ‘A number divided by 2.’ would be matched with ‘n ÷ 2’ or ‘n/2’ ‘A number multiplied by itself.’ would be matched with ‘n × n’ or ‘n2’ ‘3 times a number’ would be matched with ‘3n’.Emphasise that when we use letters to represent numbers, the multiplication sign is usually left out (e.g. 3 × *a* is written as 3*a*).* Provide learners with real-life problems for them to write as simple algebraic expressions, such as:

 Lazar is collecting tokens. Each token is worth 25 points*. How many points does Lazar have?* (e.g. 25*t*, where *t* is the number of tokens.) | Sets of cards showing a range of terms, expressions and equationsTwo sets of cards per pair:Set 1: 10 cards showing 1- or 2-digit numbersSet 2: letters or shapes representing unknown numbersTwo sets of matching cards per pair:Set 1: simple expressions Set 2: matching word statementsPre-prepared real-life problems that can be expressed as simple algebraic expressions |
| 7Ae3 | Construct simple algebraic expressions by using letters to represent numbers. |
| 7Ae2 | Know that algebraic operations follow the same order as arithmetic operations. | * Use a calculation to recap on the order of arithmetic operations, e.g. *What is 3 × 4 + 5? Why? What is 3 × (4 + 5)? Why?*

Reinforce how we can use brackets to change the order in which operations are performed.Explain that algebraic expressions are formed in a similar way to numerical expressions, but with letter symbols as well as numbers combined with operation signs. Explain that algebraic operations follow the same order as arithmetic operations too. * Ask learners to work in pairs. Give each pair a set of expression cards and a set of word cards, and ask them to match equivalent cards.
* Give each small group an algebraic expression containing several operations. Learners agree the order in which operations will be carried out. Choose a learner to present the expression and the order of calculation to the class.
 | Two sets of matching cards per pair:Set 1: expressions involving more than one operation, e.g. 3*a* – *b,* 3(*a* – *b)* Set 2: matching word statements, e.g. 3 times *a* minus *b;* Subtract *b* from *a* and multiply the result by 3 |
| 7Ae4 | Simplify linear expressions, e.g. collect like terms; multiply a constant over a bracket. | * Show objects or pictures that represent expressions in order to model collecting like terms to simplify expressions.

Begin by discussing expressions with the same variable, e.g. 8*x* + 3*x* (e.g. modelling using 8 books and 3 books) and 7*y* – 4*y.*Then move on to expressions with more than one variable, e.g. 5*a* + 5*b* + 2*a* and including a mixture of operations, e.g. 6*h* + 4*d* – 3*h*.  Finally discuss multiplying a constant over a bracket, e.g. 2(3*a* + 4*b*).* Learners use term cards to generate and simplify expressions of the form:

🞏 + 🞏 + 🞏🞏 – 🞏 + 🞏They then generate, expand and simplify, where appropriate, expressions of the form:2 (🞏 + 🞏)2 (🞏 – 🞏)Learners justify their answers to a partner.* Learners complete a ‘like terms’ pyramids, where they find the answer for each brick by adding the terms on the two bricks below. *Is your expression as simple as possible? How do you know?*

Encourage discussion between partners to compare completed pyramids. | Objects or pictures to represent unknownsCards showing algebraic terms, e.g. -3*x*, +2*x*, 5*y*, -1*y*… |
| 7As1 | Generate terms of an integer sequence and find a term given its position in the sequence; find simple term-to-term rules. | * Explain that a sequence is a set of ordered numbers that follow a rule and each number in the sequence is called a ‘term’.

Ask learners to sit in pairs. State the first four or five terms in an integer sequence, e.g. 8, 16, 24, 32, …  or  89, 80, 71, 62, 53, …Ask learners to take turns in their pairs to generate the next term in the sequence*. How did you know which term was next? What information did you use from the first terms I gave you to work it out? What is the term-to-term rule?** Learners make a spreadsheet or a simple computer program to generate terms of an integer sequence. *What is the term-to-term rule?*
* Learners play ‘Race against the clock’ in small groups. One learner takes a first term card and a term-to-term rule card. They have 30 seconds to see how many terms they can generate. For example, if they pick a starting card of 32 and they have the rule ‘double and add one’ they recite: ‘32, 65, 131, 263, 527 …’. The winner is the learner in the team who recites the most correct terms in 30 seconds.
* In pairs, learners generate sequences for their partner to identify. The partner states the first term and the term-to-term rule.
* Learners work in small groups. They take turns to generate the first four terms in a sequence of their choice and challenge others to find the 5th, 10th and 20th terms. *What is the term-to-term rule for the sequence?*
 | Spreadsheet software or suitable computer programming toolTwo sets of cards:Set 1: first term cards, e.g. 3, 56, 12, 9, 25Set 2: term-to-term rule cards, e.g. add 3, double and subtract 1 |
| 7As2 | Generate sequences from spatial patterns and describe the general term in simple cases. | * Learners use cards showing general rules. They find the first five terms of the matching integer sequences.
* Provide learners with a range of resources and ask them to construct a sequence of patterns using e.g. blocks, shapes or sticks:

matchsticks%201matchsticks%202Learners write the first five terms of the matching number sequences. They then identify the rule for the general term using their pattern. *How is this pattern constructed? What information did you use from constructing the pattern to help you to identify the general term?** Learners create a sequence of patterns and challenge a partner to identify the general term. *Is there more than one general rule? If so, why is that?*

Provide learners with a range of resources and ask them to create and continue more unusual sequences of patterns, e.g. with counters: They write the first five terms of the matching number sequence and identify the rule for the general term using their pattern. *How is this pattern constructed? What information did you use from constructing the pattern to help you with the general rule?* | Cards showing general rules for sequences, e.g.The general term is 4*n*.The general term is *n* + 4. The general term is 105 – 5*n*. Resources for making patterns, e.g. buttons, sticks (e.g. toothpicks), counters, cubes |

Problem Solving

Below are some possible activities linking the problem-solving strand to the knowledge and understanding learning objectives for the unit. To enable effective development of problem-solving skills, you should aim to include problem-solving opportunities in as many lessons as possible across the unit.

| **Activity ideas** | **Resources** |
| --- | --- |
| Learners work in pairs to sort three sets of cards: a description of a situation, a measurement and a unit.  | Three sets of sorting cards:Set 1: descriptions of measurement situations, e.g. temperature of a freezer, cooking time for a cakeSet 2: a numerical measurement, e.g. -20, 45Set 3: a unit, e.g. degrees, minutes |
| Strips of paper 30 cm long are joined together to make a longer streamer.The strips overlap by 5 cm.Learners investigate different length strips and lengths, e.g.  How long is a streamer made from two strips? How many strips does a streamer 280 cm long require?5cm30cm30cm5cm |  |
| Learners carry out real-life measures investigations, e.g. * A tap drips ¼ ml of water every second. If the tap was dripping non-stop for one hour … one day … one week … one month, how much water would be wasted?
* It is possible to reduce the amount of water that is used when you flush a toilet by placing a brick into the cistern. How much water would one brick save in 1 flush? … 5 flushes? … 10 flushes? … 100 flushes? Is there a general rule or formula that can be used?
 |  |
| Ask learners to think of a number and write it down. They: * Add 10.
* Double the answer.
* Subtract 6.
* Halve the answer.
* Take away the original number.

What do learners notice about their final answers? (Everyone gets 7.) Learners work in pairs or small groups to use their knowledge and understanding of algebra to work out why everyone gets the answer 7. Establish that the expression for the starting number *x* simplifies to 7, which is why everyone gets that answer: 2(*x* + 10) – 6 *– x* 22*x* + 20 – 6 – *x*22*x* + 14 *– x* 2 *x* + 7 – *x* 7You could challenge learners to make their own ‘think of a number’ trick. |  |
| Learners complete ‘like terms’ pyramids with a range of missing expressions, e.g. *How can you find this missing expression? How can you use inverse operations to help you?*Learners then make up their own 'like terms' pyramids for a partner to complete. *Is it possible to complete all of your missing expressions? Why?*Encourage discussion between partners to compare completed pyramids. |  |
| Here is a continuing pattern of white and blue tiles. Pattern 1 Pattern 2 Pattern 3Learners explore this pattern to find the number of tiles in Pattern 8. They give an explanation of how they worked it out. Learners decide whether any of the patterns will have 25 white tiles, giving a mathematical reason for their answer.  | Square tiles if needed |
| Learners explore given linear sequences. They find a method to give the *n*th term and use this to calculate a given term, e.g.* First term is 6, term-to-term rule is ‘Add 3’. Find the 50th term.
* First term is 200, term-to-term rule is ‘Subtract 10’. Find the 20th term.
* A truck transports 800 kg of fruit on its first journey, 950 kg on its second journey and 1100 kg on its third journey. If the sequence continues in the same way, how much fruit does the truck transport on its 6th journey? How much fruit has the truck transported in total after 10 journeys?
 |  |
| Learners explore sequences where they don’t know the first term. They to find a method to give the *n*th term and use this to calculate a given term, e.g. * The term-to-term rule is ‘Add 10’ and the 6th term is 52. *Find the 15th term … 1st term.*

In pairs, learners explain their method to one another. |  |

Unit 1C: Handling Data, Geometry and Problem Solving

Handling Data and Geometry

| Curriculum framework codes | Learning objectives | Activity ideas | Resources |
| --- | --- | --- | --- |
| 7Gs1 | Identify, describe, visualise and draw 2D shapes in different orientations. | * Review and revise previous knowledge of 2D shapes. Ask learners to work in pairs, and give each pair a set of shape cards and a set of name cards. Ask them to match the name and the shape.
* Ask learners to sketch a triangle on their mini whiteboards, then another, and another and another. *How many different triangles can you draw? Are these two* (with different orientations) *the same or different? Why? What is the same … different about these two triangles?*
* In groups, learners sketch and name as many different straight-sided 2D shapes as they can. This is a useful task to assess learners’ knowledge of 2D shapes.
* Learners close their eyes and visualise different 2D shapes. Ask them to imagine acting upon the shapes in different ways, e.g.

 Visualise a square. Draw a vertical line through the square. *What two shapes do you have now?* (Isosceles right-angled triangles, rectangles … )Ask individual learners to come to the board and sketch their shape and the straight line and discuss with the class the different possibilities. Visualise a rectangle. Draw a diagonal line between two opposite corners. *What shapes have been created now? How specific can you be when naming the triangles?* (Scalene right-angled triangles.) | Sets of name cards, e.g. quadrilateral, scalene triangle, square; and sets of shape cards to matchMini whiteboards and pens  |
| 7Gs2 | Use the notation and labelling conventions for points, lines, angles and shapes. | * Learners draw a line segment, AB, that is a given length. Check for correct labelling (and using capital letters). *What are you identifying when you label a line segment AB? Can you show me point A and point B on your line segment?*

 Next ask learners todraw a point C away from the line segment. Ask them to draw and mark angle ABC. * Model the convention $∠$ABC for identifying angles. Ask learners to write this, paying careful attention to the symbol having a horizontal base line so it doesn’t look like an inequality sign.
* Learners sketch a range of shapes, labelling their vertices and marking the angles. *How do we mark right angles differently from other angles?*

 Highlight the convention of identifying shapes using their vertex labels, e.g. a triangle ABC.* Give learners a range of points, line segments, angles and shapes. Ask them to label and/or to identify particular parts of each one.
 | Pre-prepared sheets with points, line segments, angles and shapes |
| 7Gs3 | Name and identify side, angle and symmetry properties of special quadrilaterals and triangles, and regular polygons with 5, 6 and 8 sides. | * Learners work in groups of 2–4 with a set of pre-constructed quadrilaterals and triangles. Learners discuss what they notice (or already know) about the side, angle and symmetry properties.
* Give learners 2D shape property statements on cards. In pairs or small groups, learners list all the shapes they can think of that have each property. *Have you included all possible shapes? Which shape appears most often in your lists? Why do you think that is? Which shape appears least often? Why do you think that is? Are there any shapes that do not have any of these properties? Can you draw one? What properties does it have?*
* Give learners cut-out cardboard shapes. Learners sort the shapes in different ways: by symmetry, angles, number of sides…
 | Sheet with quadrilaterals and triangles (e.g. square, trapezium, kite, equilateral triangle)2D shape property statements on cards, e.g. ‘It has two equal and opposite pairs of angles.’Cut-out cardboard shapes |
| 7Gs4 | Estimate the size of acute, obtuse and reflex angles to the nearest 10°. | * Discuss acute, obtuse and reflex angles and the need to check which scale on the protractor to use.*What are degrees used to measure? What is special about acute angles … obtuse angles?* Introduce the term 'reflex angle' to describe an angle of between 180° and 360°.
* Show a range of angles. Learners work in teams to estimate each angle to the nearest degree. If teams are correct to a specified level of accuracy (e.g. to the nearest two degrees), they are awarded one point. The winning team is the one with the most points at the end.
* Challenge learners to use only a ruler to draw an angle of a certain size. A partner checks the size of the angle with a protractor. Award points for drawing within five degrees of accuracy (either way) and five bonus points for getting the exact angle.
 |  |
| 7Dc17Dc2 | Decide which data would be relevant to an enquiry and collect and organise the data.Design and use a data collection sheet or questionnaire for a simple survey. | * Learners decide on a topic for data collection, giving a sound reason for their choice. It may relate to a topic they are studying in school, or something that is personally interesting to them.

They identify a research question or hypothesis to investigate. *What is the difference between a research question and a hypothesis? Why have you chosen a research question rather than a hypothesis? Why will your research question / hypothesis be interesting to investigate?* Possible topics might be *What makes an average learner in our class?*  Learners could collect data such as height, how far away from school learners live, interests or ambitions. (Note: Remind learners to consider others' sensitivities when deciding what data to collect.) They decide how they will calculate an ‘average’.  *The environment* Learners could collect data on recycling, costs of electricity and/or gas or ways that people travel to school. *Sport*  Learners could ask questions like 'Does the population of a country impact on the number of medals they win at the Olympics?' or 'In which athletic events are womenclosest to men in terms of the world record?'  Use questioning to assess whether groups have thought about everything they need to, or to suggest new ideas. *How will you collect your data? How much data will you be able to collect? Will your data be discrete or continuous? How will you record your data? Will you have to organise your data before you can present it? What diagrams, charts and graphs are suitable for presenting your data? Why?* Learners begin to think about how they will collect the data they need and how to organise and present it. Learners design their data collection sheets that will be used later. They consider the possible bias in different situations and questions. They should consider the possible bias in different questions, and how to use pre-selected options effectively. *Is this a good question? Why? Do your questions cover all possible options?*Learners create and use their data collection sheet. They then reflect on the effectiveness of their design. **Note:** The Stage 7 learning objectives relating to presenting and interpreting data are focuses of Units 2C and 3C. However, you may decide to provide learners with the opportunity to use their existing data handling skills to present and interpret the data they have collected in this unit. |  |
| 7Db17Db2 | Use the language of probability to describe and interpret results involving likelihood and chance. Understand and use the probability scale from 0 to 1. | * Recap the language of probability and the probability scale (using words):

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Impossible | Unlikely | Evens | Likely | Certain |

 Provide statements for learners to place along a probability scale. * Give each learner a mini whiteboard and pen. Read out a statement (e.g. It will rain tomorrow) and ask the learners to write a word of their choice to describe how likely it is that this event will occur (e.g. almost certain, highly likely, very unlikely). Read out some of their contributions and discuss.*Is ‘highly likely’ the same as ‘almost certain’? Why? Is one term better than the other?*
* Divide learners into five groups. Give each group a large sheet of paper and some marker pens. Give each group a different likelihood word to write in the middle of their sheet, e.g. ‘Impossible’, ‘Unlikely’. Each group records events to match their word on their sheet (e.g. If the word is impossible, they might write ‘Our teacher will go to the moon tomorrow.’ or ‘My favourite football team will score a million goals in one game.’). When they have finished, ask learners to move around and look at what other groups have done, challenging any events they think are on the wrong sheet.
* Show a probability scale with ends labelled 0 and 1. Discuss the numerical probability of different likelihoods on the scale:

 Impossible = 0 Unlikely ≈ ¼, 0.25 or 25% Evens = ½, 0.5 or 50%  etc. Establish that: probabilities can be expressed in decimals, fractions or percentages Stress that exactly where unlikely and likely fit on the probability scale is a matter of opinion, so the numerical values for these are approximate.* Ask learners to give appropriate likelihood language to match a range of given probabilities, e.g. 0.1. Discuss differences of opinion.

Then discuss where given events belong on the 0–1 probability scale and their estimated numerical probability. | A large probability scale per groupOne set of statements per group, e.g.* It will be sunny tomorrow.
* We will all be at school tomorrow.
* The news will broadcast on television tonight.
* One of us will travel to Mars next week.
* Some of us will change our clothes this week.

Mini whiteboards and pensLarge sheets of paperMarker pensA large probability scale with ends labelled 0 and 1A large probability scale with ends labelled 0 and 1 |
| 7Db3 | Find probabilities based on equally likely outcomes in simple contexts. | * Give each group a coin, a 6-sided dice, a numbered spinner, and a bag of differently coloured objects.

*What are the possible outcomes when we toss a coin?* (Number side or non-number side). *What is the probability of each outcome?* Establish that each outcome is equally likely (an even chance), so the probability of each outcome is ½ or 0.5 or 50%. *What do you notice about the sum of the probabilities? Why is this?* (The sum is 1 because the likelihood of getting either one outcome or the other is certain.)Discuss the probabilities related to a given possible outcome when you throw a dice, spin a spinner or take a coloured object from a bag.  | Coins6-sided diceNumbered spinnersBags of differently coloured objects |

Problem Solving

Below are some possible activities linking the problem-solving strand to the knowledge and understanding learning objectives for the unit. To enable effective development of problem-solving skills, you should aim to include problem-solving opportunities in as many lessons as possible across the unit.

| **Activity ideas** | **Resources** |
| --- | --- |
| Show learners 2D shapes sorted according to their properties. *Why are these shapes grouped together? What is the same … different about them? Draw three more examples of shapes that would belong in this group.* | Pre-prepared groupings of shapes |
| Challenge learners to construct (rather than draw) named 2D shapes using basic geometry software.  | Geometry software |
| Two squares can be overlapped to make different shapes, e.g. a rectangle and a pentagon.Learners decide which of these shapes can be made in this way:* rhombus
* isosceles triangle
* hexagon
* octagon
* kite
* trapezium.

If a shape cannot be made they explain why. Learners work with squares that are the same size or different sizes. | Squares of different sizes |
| Give learners some information about a quadrilateral:* Two angles are equal.
* The third angle is equal to the sum of the two equal angles.
* The fourth angle is 60° less than twice the sum of the other three angles.

Learners find the sizes of the angles in the quadrilateral. |  |
| Learners work in pairs to decide whether each statement below is always, sometimes, or never true.If they are sometimes true, learners give examples or conditions when are true and when they are false. If they are always true or never true, learners must give convincing reasons as to why that is the case.* In any school, there will be two people who share a birthday.
* A randomly selected person from New York will live to a greater age than a randomly selected person from Delhi.
* If a learner rolls a die 100 times, he will get about the same number of 1s as 6s.
* If a learner flips a fair coin 20 times, it will land heads 10 times.
* Jane takes part in a true of false quiz. She doesn’t know any of the answers so she guesses. It is certain that she will get 5 answers correct out of 10.
* If you toss a coin twice, the probability of getting exactly two heads is ½.
* In a football (soccer) match, a team can win, lose or draw. The probability of losing is therefore 1/3.
* If a fair coin is tossed 10 times and you get 10 heads, the next time the coin is tossed you are more likely to get a tail than a head.
* If you choose four letters from the alphabet at random, you are more likely to get F, J, M and P than A, B, C and D.

Learners could work together to adapt any of the statements that are sometimes true to make them always or never true. |  |
| Kay chooses numbers 1, 2, 3, 4, 5, and 6 from a pack of 1–200 number cards.Zak chooses 14, 45, 76, 137, and 182.Mary then says a number at random from 1 to 200.Learners decide whether Kay or Zak is more likely to have Mary’s number. They give mathematical reasons for their answer. |  |

Unit 2A: Number, Calculation, Measure and Problem Solving

Number, Calculation and Measure

| Curriculum framework codes | Learning objectives | Activity ideas | Resources |
| --- | --- | --- | --- |
| 7Ni2 | Recognise multiples, factors, common factors, primes (all less than 100), making use of simple tests of divisibility; find the lowest common multiple in simple cases; use the ‘sieve’ for generating primes developed by Eratosthenes. | * Use the Sieve of Eratosthenes to identify prime numbers. As learners are identifying multiples of different numbers, remind them about the tests for divisibility covered earlier in the year: *How do you know that this number is a multiple of …?*
* Ask learners to state as many multiples of a given numbers as possible in 30 seconds. Repeat the activity in the next lesson, challenging learners to improve their score.
* Recap lowest common multiples. Give the learners six numbers (e.g. 4, 6, 9, 10, 12, 15). Ask them to list the multiples of each. Now choose any two of the numbers and ask the learners to identify the lowest multiple they have in common. *When is the lowest common multiple equal to the product of the two numbers?*
* Give each small group of learners a set of 2–100 number cards. They group the numbers into three sets according to how many factors they have: ‘Exactly two factors’, ‘An even number of factors greater than two’, ‘An odd number of factors’. *What do you notice about the numbers in each set?* (Identify the prime numbers, square numbers and rectangular numbers). *Can you find numbers within each group with common factors?*
* Have a 'Factor race' in teams. Say a number. Teams race to be the fastest team to write a complete factor list.
 | 100 square Coloured pencils A set of 2–100 number cards for each group |
| 7Ni3 | Recognise squares of whole numbers to at least 20 × 20 and the corresponding square roots; use the notation 72 and $\sqrt{49}$. | * Use squares drawn on a grid to remind learners about the meaning of a square number. Model using the notation 2 and √.
* Learners play a matching card game in groups of 3 or 4. They start with cards arranged face down. They take turns to turn over two cards. If the cards match (e.g. √9 and 3 or 112 and 121), the learner keeps their cards, explains how they know they are a pair, then has another turn. If the cards do not match, they are returned to their positions face down. The winner is the learner who collects the most pairs.
* Learners work in small groups. They place the number cards into a bag. They take turns to take a number from the bag and state if it is a square number or not. If the rest of the group decides they are correct, they keep the card. If they are incorrect, the number goes back into the bag. The bag is passed around until there are no numbers left. The winner is the learner with the most cards. To make the game more challenging, give learners a time limit for their decision.
 | Set of cards for each group: √1, √4, √9, √16, √25, √36, √49, √64, √81, √100, √121, √144, √169, √196, √225, √256, √289, √324, √361, √400, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 12, 22, 32, 42, 52, 62, 72, 82, 92, 102, 112, 122, 132, 142, 152, 162, 172, 182, 192, 202, 1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144, 169, 196, 225, 256, 289, 324, 361, 400Number cards (all the square numbers from 1 to 400 as well as prime numbers and other rectangular numbers, such as: 5, 12, 50, 66, 204, 250 …)Opaque bag (or box) for the cards to be drawn from(Optional) Timer |
| 7Nf4 | Add and subtract two simple fractions e.g. 1/8 + 9/8, 11/12 – 5/6; find fractions of quantities (whole number answers); multiply a fraction by an integer. | * In pairs, learners discuss an addition of two fractions with the same denominator with a total greater than 1, e.g. 1/8 + 9/8. They create diagrams to justify their answers using the circles or rectangles provided. *Is your answer in its simplest form? How do you know?* Repeat for an addition of fractions with different denominators, e.g. 2/3 + 5/8. *Will the answer be greater than 1? Why? How do the diagrams help you to calculate the answers?* (Establish that it effectively converts the fractions into fractions with a common denominator.)
* Learners use the paper circles or squares to create and answer their own additions and subtractions of fractions. *How are you deciding which fractions your circles/rectangles will help you to calculate with?* (Fractions that can be converted to twenty-fourths.)
* Discuss adding and subtracting fractions with different denominators, e.g. 11/12 – 5/8. *What diagram could we use to help us to calculate the answer? How could we calculate without a diagram?* Model converting to fractions with a common denominator to calculate. *How can we approximate the answer before we calculate?*
* Learners use fraction cards to generate additions and subtractions of fractions. *Do you need to convert both fractions to find the answer? Why?*
* Learners work in pairs to create a list of instructions for calculating a fraction of a quantity, e.g. 3/8 of 240. Discuss and refine the instructions as a class. *How can you check whether your answer is reasonable?* (e.g. It should be between ¼ and ½ of 240, so between 60 and 120.)
* Give learners quick-fire fraction of quantities questions, e.g. 7/8 of 32. They show their answers on mini whiteboards.
* Show a fraction of a quantity, e.g.4/5 of 155. *Will the answer be a whole number? Why?* (Yes, because 155 is divisible by 5.)

Learners apply their understanding of tests of divisibility to make up their own fraction of quantity questions with whole-number answers*. How do you know the answer is a whole number?* They share their questions with a partner and compare answers and discuss any discrepancies.* Show a multiplication of a unit fraction by an integer, e.g. 1/3 × 6. Establish that this can be thought of as 6 lots of 1/3 or as 1/3 of 6.

Model both interpretations using pictures of pizza (6 thirds of pizza or 6 pizzas divided into 3 groups).In pairs, learners discuss how they could find, e.g. 2/3 × 7. Discuss and model using pizza pictures. Establish that 2/3 × 7 = 14/3 = 42/3.* Learners use dice to generate multiplications of the form:

$$\frac{}{} ×$$They calculate their answer and then use their knowledge of converting fractions to decimals using a calculator to check that their answers are reasonable (e.g. for 2/3 of 34, keying in 2 ÷ 3 × 34 = and comparing 22.6666... with their fraction answer).* Write a set of possible answers to multiplications of fractions by integers on the board, e.g. 2/3, 13/5.Give one question at a time (e.g. 1/3 × 2; 2/5 × 4). Learners choose from the answers on the board and write their choice on their mini whiteboard.
 | Paper circles or rectangles divided into 24 equal parts Sets of fraction cardsMini whiteboards and pensPictures of pizzas or similar to model fraction multiplications DiceCalculatorsMini whiteboards and pens |
| 7Nf5 | Understand percentage as the number of parts in every 100; use fractions and percentages to describe parts of shapes, quantities and measures. | * Discuss the etymology of ‘percent’ – from Latin ‘per centum’ meaning ‘for every hundred’. Explain that in some contexts a percentage can be more than 100, e.g. if a company’s profits have gone up by 16% this year, their profits are 116% of last year’s.
* Learners divide squares/rectangles with different areas drawn on squared paper into different percentages, e.g. a 20 × 10 rectangle, 25% red, 30% green, 45% blue.
* Show learners a picture of a 500 ml measuring jug filled to 100 ml. Ask them what percentage of the jug is filled (20%). Discuss. Now show them more pictures of objects with a percentage indicated (e.g. a square with 25% shaded, a point 60% along a metre stick) and ask them to give the percentage.
* Ask learners to indicate different percentages of a range of shapes, quantities and measures. Include percentages over 100%, e.g.

Pour water into a 100 ml jug so it is 40% full. Mark a point 67% along a 1 m line segment.  Using 10 × 10 squares, colour 124%.  Discuss how learners found the different percentages. *How did you know that was … %? What does percent mean? Is there another way you could have found that percentage? What would this percentage look like as a fraction? … as a decimal?** Learners take a percentage card out of the bag and write it as a number of parts per one hundred, e.g.

60% is equivalent to 60/100 150% is equivalent to 150/100.  Challenge learners by also asking them to find the equivalent decimals, e.g.  60% is equivalent to 60/100 and 0.6; 150% is equivalent to 150/100 and 1.5. | Squared paperColoured pencilsVariety of shapes, quantities and measures that learners can use to find percentages, e.g. 100 ml containers, metre rulers, 10 × 10 squares, lumps of dough and weighing scales, tubs of dried peas, paper circlesCards showing percentages, including some greater than 100%Opaque bag (or box) |
| 7Nc5 | Calculate simple fractions and percentages of quantities, e.g. one quarter of 64, 20% of 50 kg. | **Note:** Fractions of quantities are addressed in 7Nf4 above.* Establish that:

10% is equivalent to 1/10 = 0.1and  5% is half of 10%.  Ask learners to use this information to find percentages mentally, e.g. 10% of $20 (by dividing by 10) 10% of 37 g (by dividing by 10) 5% of $5 (by finding 10% and halving) 100% of 4 litres (by knowing that 100% represents the whole) 15% of 40 (by finding 10%, then 5% and adding the results together).  Learners write their answers on mini whiteboards. *How did you find the answer? What strategy did you use? How did you use your knowledge of percentages, fractions, decimals and multiplying/dividing by 10?** Explain that 1% is equivalent to 1/100 = 0.01. Learners use mini whiteboards or paper to help them calculate percentages of larger numbers, e.g.

11% of $ 2800 70% of 130 g.* Using a calculator and without using the percentage key, learners calculate percentages, e.g.

24% of 34  14.5% of 56 litres.* Combine learning on percentages of quantities with earlier learning on fractions of quantities.

Give learners three sets of cards: answers, fractions of quantities and percentages of quantities. Learners work in pairs to match the cards (e.g. 4 with ¼ of 16, 2/3 of 6, 50% of 8). Note that each group could have two, three or more cards. | Mini whiteboards and pensMini whiteboards and pensCalculatorsThree sets of cards: Set 1: answers (e.g. 4)Set 2: fractions of quantities (e.g. ¼ of 16, 2/3 of 6)Set 3: percentages of quantities (e.g. 25% of 16) |
| 7Nc6 | Use the laws of arithmetic and inverse operations to simplify calculations with whole numbers and decimals. | * Give learners an integer addition to solve mentally, e.g. 1589 + 446. Ask them to model how they worked out the answer. Discuss different methods (e.g. partitioning into thousands, hundreds, tens and units; rounding and adjusting). *Are there any other methods that could have been used? Which might be the most efficient? Why? When would a different method be more efficient? How does our place value knowledge help us to quickly work this question out mentally?*
* Give learners an integer subtraction to solve mentally, e.g. 2007 – 1998. Ask them to model how they worked out the answer. Discuss different methods. *How can addition (counting up) be used to carry out subtractions?*
* Give learners a multiplication to solve mentally, e.g. 25 × 12. Ask them to model how they worked out the answer. Learners compare methods of performing the calculation. *Can more than one method get the correct answer? How, why, when?*Discuss different methods. *Are there any other methods that could have been used? Which might be the most efficient? Why? When would a different method be more efficient?*
* Give learners a division to solve mentally, e.g. 3 ÷ 12. Ask them to explain how they worked out the answer. Discuss different methods. *How can multiplication be used to help solve division calculations?*
* Demonstrate using the grid method of multiplication for partitioning to find the solution, e.g. for 23 × 45:

  20 3 40 5

|  |  |  |
| --- | --- | --- |
|  | 20 | 3 |
| 40 | 800 | 120 |
| 5 | 100 | 15 |

23 × 45 = 800 + 100 + 120 + 15 = 1035 * Learners work in groups of 2 or 3 to answer a series of mental additions and subtractions with decimals. Include questions where there is a different number of digits in each number. Learners may use jottings if they wish. The main aim of this task is for learners to discuss their ideas, not for speed. *How did you add/subtract these numbers mentally? What mathematical knowledge and skills were you using as you found the solution? How can you check your answers?*
 |  |
| 7Ml3 | Read the scales on a range of analogue and digital measuring instruments. | * Provide a range of objects for learners to measure the lengths, masses and capacities of and a variety of measuring instruments. Discuss how learners decided how to read the different scales and how they ensured accuracy in their measurements.
* Draw learners’ attention to a range of different measuring scales, e.g. rulers; a dial on kitchen scales to measure mass; a clock face to measure the passing of time; the scale on a measuring jug to measure capacity. *What is the same and different about these scales?* Clarify that a clock uses a different base system (60 minutes = 1 hour) and that although they look similar it is important to note what is being measured (e.g. length, mass, duration, angle).
* Discuss how analogue weighing scales have different scales for different purposes. Compare digital and analogue scales. *Which type of scale do you prefer reading? Which do you think is more accurate? Why?*
 | Measuring instruments with a variety of different scalesObjects to measure |

Problem Solving

Below are some possible activities linking the problem-solving strand to the knowledge and understanding learning objectives for the unit. To enable effective development of problem-solving skills, you should aim to include problem-solving opportunities in as many lessons as possible across the unit.

| **Activity ideas** | **Resources** |
| --- | --- |
| Secretly write down a number less than 100. Say one of its factors and ask learners to say what the number could be and why. Give two factors. *Could your answer still be correct? Why?* Repeat until your secret number is the only remaining possibility. |  |
| Learners investigate problems involving factors and multiples e.g.* Two lighthouses are visible from shore. They both flash at 8:30. If the first lighthouse flashes every 45 seconds and the second flashes every 1.25 minutes, when will be the next time they both flash at the same time?
* Three runners jog on a circular track. The first takes 45 seconds to complete one lap, the second takes 55 seconds, and the third takes 1 minute. If they start at the same time, when will be the next time they are all together at the starting position?

With the same information as above, when will be the first time all three runners are at the same point on the track, whether or not it is the starting position?  |  |
| Find the smallest number with exactly three factors … four factors. What about numbers with different numbers of factors?Learners investigate explaining their approach and results, and generalising wherever possible.  |  |
| Margaret says that *a, b* and *c* are prime numbers and 720 is the same as *a*4× *b*2 × *c*. What are the values of *a, b* and *c?* |  |
| Mrs Jones buys some potatoes and carrots. 65% of the total mass is carrots. There are 2.7 kg less potatoes than carrots. How may kg of potatoes does she buy? |  |
| Explain that unit fractions have numerators of 1.Show1/2 = 1/3 + 1/6 Ask learners to jot down anything they notice*. Is this addition correct?*Give other examples of unit fraction additions:1/2 = 1/10 + 1/201/3 = 1/4 + 1/121/3 = 1/7 + 1/211/4 = 1/5 + 1/20*Are all these additions correct?* Ask learners to say what they notice about the sums which are correct.Learners work in pairs to generate some other pairs of unit fractions with totals which are unit fractions. They explain to each other how to generate lots of correct examples. |  |

 – Stage 7 Mathematics

Unit 2B: Algebra, Measure and Problem Solving

Algebra and Measure

| Curriculum framework codes | Learning objectives | Activity ideas | Resources |
| --- | --- | --- | --- |
| 7Ae5 | Derive and use simple formulae, e.g. to change hours to minutes. | * Ask learners some quick-fire questions related to conversion between two specific units (e.g. hours to minutes), each time asking learners how they worked out the answer. Note the questions and answers on the board. *What do you notice?* Establish that, regardless of the starting number, it is always multiplied by a constant (i.e. the direct proportion multiplier), e.g. for hours to minutes, the hours are always multiplied by 60. *How could we write that as a formula?* (e.g. *m* = 60*h*)
* Learners create formulae using spreadsheets to convert from one set of units to another.
* Learners identify formulae to convert from one metric unit to another, e.g. from kg to g, from cm2 to m2. *Will the constant be greater than or less than 1? Why? Which conversions have the same constant?* (e.g. kg to g and l to ml)
* Give learners a statement. They derive a formula from it, e.g.

 On average, people sleep 7 hours each day. *How would I work out the hours slept in 1 week? … 1 month? … 1 year? …* n *days?* | Spreadsheet software |
| 7Ae6 | Substitute positive integers into simple linear expressions/formulae. | * In pairs, learners use 100 squares to support them in solving expression and equation questions, e.g.

 What is the value of the expression *x* + 25 when *x* = 5? What is the value of *x* when *x* + 17 = 25? *How did you find the answer? How could you find the answer if you didn’t have the 100 square?** Give the learners a mini whiteboard and pen. Display a formula, e.g. *m* = 60*h* for converting a number of hours into minutes. Ask learners to work out the minutes if *h* = 2 and to write their answer on the mini whiteboard. Repeat for different values of *h*. Repeat for different formulae.
 | 100 squares Mini whiteboards and pens |
| 7Ae7 | Construct and solve simple linear equations with integer coefficients (unknown on one side only), e.g. 2*x* = 8, 3*x* + 5 = 14, 9 – 2*x* = 7. | * Establish that some equations can be solved by simply thinking about the answer, e.g. for *x* – 3 = 5, think ‘What number, when you subtract 3, gives 5?’.

But for others it may be necessary to manipulate the equation. Discuss how operations can be applied to both sides of an equation to form an equation of the form *x* = …, e.g. For 2*x* – 1 = 5: (adding 1 to both sides) 2*x* = 6 (dividing both sides by 2) *x* = 3 Give learners lots of practice in solving simple linear equations. Start by asking learners to work in pairs so they can discuss the process. Encourage learners to record the operations they apply to both sides of the equation.* Learners write and solve equations corresponding to given word problems, e.g.
* Nevena thinks of a number. When she doubles the number and subtracts 5 the answer is 23. What is her number, *n*?
* In a class of 30 learners, there are four more boys than girls. How many girls (*g*) are there? How many boys are there?

*Why does your equation match the word problem? How can you solve it? How can you check your answer?* (By substituting it back into the equation.)* In pairs, learners discuss a word problem with two unknowns, e.g.

Tatjana goes to a café in Paris with a friend. She buys two hot chocolates, and one pastry to share. She pays €6.10. *What equation represents this problem? Can you solve it?* (No.) *What possible answers are there for the price of the pastry?* After some thinking time, clarify that the matching equation is, e.g. 2*c* + *p* = 610 (or 6.10)Establish that you cannot find a single solution to this equation because there are two unknowns. Discuss learners’ possible answers for the price of the pastry.*What extra information do you need to find the price of the pastry?* (The price of a hot chocolate.) *If one hot chocolate costs €1.75, what is the price of the pastry?*With learners' help, model finding the solution: 2*c* + *p* = 610  350 + *p* = 610 (substituting for *c*) *p* = 260 (subtracting 350 from both sides of the equation)So the pastry costs €2.60.* Provide groups of learners with cards showing variables and constants. They use the cards to construct different equations, e.g.

 3*p* + *p* + 2 + *p* = 22They work together to solve the equations, recording their workings clearly. *How can you check your answers?* | Pre-prepared word problemsCards showing variables and constants3pp2p22 |
| 7As3 | Represent simple functions using words, symbols and mappings. | * Display a function machine, e.g.

|  |  |
| --- | --- |
| **Input*****x*** | **Output*****y*** |
| 36 | 72 |
| 28 | 56 |
| 47 | 99 |

*What do you need to do to* x *to get* y*? How can we write a formula to tell us what to do to* x *to get* y*?* (*y* = 2*x.*) *What will* y *be if* x *is 25? …0? … -10?* Repeat for a different function machine. * Explain that functions map an input onto outputs. Learners draw mapping diagrams for given functions, using integer values for *x*, e.g. *x* 🡪2*x* + 1:

0 1 2 3 4 5 6 7 8 9 0 1 2 3 4 5 6 7 8 9  *How can you describe the function in words?* They use the mapping diagrams to complete tables of values for *x* and f(*x*), e.g.

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| *x* | 0 | 1 | 2 | 3 | 4 | 5 |
| f(*x*) |  |  |  |  |  |  |

 Learners carry out the activity above using negative integer and fractional values for *x*.* In pairs, learners discuss mapping diagrams and identify the functions. *How did you decide on that function? How would you describe the function in words? What would the function look like if …?*
 | Large sheets of paper Pre-prepared functions |
| 7As4 | Generate coordinate pairs that satisfy a linear equation, where *y* is given explicitly in terms of *x*; plot the corresponding graphs; recognise straight-line graphs parallel to the *x*- or *y*- axis. | * Remind learners about plotting coordinates. Display a large coordinate grid on the board, and give each learner a coordinate card. Ask each learner to come to the grid and mark their point with a cross. Other learners check the position.
* With learners' help, create a mapping for a simple linear function, e.g. *x* 🡪3*x*.

|  |
| --- |
| 1 🡪 32 🡪 63 🡪 94 🡪 125 🡪 15 |

 Relate the function to the equation *y* = 3*x*. With learners' help, create a table of values for *x* and *y*:

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| *x* | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
| *y* | 0 | 3 | 6 | 9 | 12 | 15 | 18 |

 Ask learners to use the *x* and *y* values as coordinate pairs to plot the graph, e.g. (3, 12) (2, 8) (1, 4) Reinforce what the graph represents*. What is the equation of this graph? Why?* * Give learners the equations of lines parallel to the *x*- or *y*- axis, e.g. *x* = 4, *y* = -3, *x* = 0, *y* = 0. They draw each of the lines as graphs and describe them. *What points are on your line? Why?*Discuss findings. *Without drawing the graph, what can you say about the graph y = 5?*

(It's parallel to the *x*-axis and 5 units above it.) *… x = -2?*  | Large coordinate gridOne coordinate card per learner – all four quadrants, e.g. (1, 2), (-3, 4), (-2, -3), (4, -1)Grid sheets with pre-drawn horizontal and vertical lines |
| 7Ma1 | Know the abbreviations for, and relationships between, square metres (m2), square centimetres (cm2) and square millimetres (mm2). | * Recap the abbreviations for square metres (m2), square centimetres (cm2) and square millimetres (mm2). *What do we use these units to measure?* State an object (e.g. pinhead, football pitch) and ask learners to show a card with the most appropriate unit to use to measure the area of the object. *What do you estimate the area to be? Why?*
* In small groups, learners make a square metre and a square centimetre, and try to make a square millimetre. Compare the sizes by asking a range of questions such as: *How many cm2 are in 1 m2? How many mm2 are in 3 cm2?* Learners use diagrams to help them to calculate answers.
* In small groups, learners group equivalent area measurements in piles of three cards.
 | Three cards per learner, each showing m2, cm2 or mm2or Mini whiteboards and pensMetre rulers30 cm rulers showing cm and mmStringScissorsPaper for cutting upSets of cards showing pairs of equivalent measurements using square metres (m2), centimetres (cm2) or millimetres (mm2) on different cards |
| 7Ma2 | Derive and use formulae for the area and perimeter of a rectangle; calculate the perimeter and area of compound shapes made from rectangles. | * *How are area and perimeter different?* Present learners with a range of rectangles drawn on squared paper. Ask them to write the length, the width, the area and the perimeter. *How can you find the perimeter using the squared paper … without using the squared paper? What about the area?*
* Recap the formulae for areas and perimeter of a rectangle. *Why do these formulae work?* Use diagrams to demonstrate.
* Challenge learners to draw as many rectangles as they can with an area of 24 cm2. You could add challenge: *Can you make rectangles with decimal lengths?* (e.g. 2.5 cm x 9.6 cm). *What are the dimensions of the rectangle with longest … shortest perimeter? How do you know you have found it?*
* Remind learners how to divide compound shapes into rectangles, then using the formula find the area of each and the total area.

 Learners solve problems to calculate the perimeter and/or area of compound shapes that can be split into rectangles. Learners compare and discuss their answers with a partner. *How will you split the shape? Do you have all the side lengths you need? How can you calculate any extra ones you need? Is there only one way to split the shape?* | Pre-prepared compound shape problems (diagrams not-to-scale) |
| 7Ml2 | Know abbreviations for and relationships between metric units; convert between:* kilometres (km), metres (m), centimetres (cm), millimetres (mm)
* tonnes (t), kilograms (kg), and grams (g)
* litres (l) and millilitres (ml).
 | The activities below are in the context of length. They can be adapted for mass and capacity.* Check that learners know the abbreviations for kilometres (km), metres (m), centimetres (cm) and millimetres (mm) (and decimetres, dm, if appropriate to your context). Also check learners are confident that they would know when to use each of these units.
* Learners investigate patterns when converting a larger unit of length to a smaller one and vice versa, e.g.

 - changing 36 cm into mm - changing 0.89 km into m - changing 0.56 m into mm - changing 3 cm into m - changing 4 mm into cm.*What do you notice?* Link back to multiplying and dividing by 10, 100 and 1000. Also note that when converting to a smaller unit, the number will be greater, and when converting to a larger unit, the number will be smaller.* Learners play ‘Snap’ in pairs. One learner deals the cards. Learners takes turns to put one of their cards face up in the middle of the table. If the card is equivalent to the last card placed, then the first learner to place their hand on the pile and shout ‘snap’ wins all the cards on the table. The winner is the first learner to collect all the cards.
* Put a sticker showing a length on each learner’s back (learners are not allowed to know what their sticker says). Learners match their peers to others who have equivalent lengths on their stickers. Learners ask each other questions to predict what the length on their sticker might be (*Is my sticker longer than my foot? Is my sticker shorter than your pencil? Is my sticker longer than the board?*).
 | A set of cards per pair, showing equivalent lengths, e.g. six of the cards might be:2000 m, 2 km, 200 000 cm83 cm, 0.83 m, 830 mm(Include three or four equivalent lengths to improve matching.)Stickers showing a length (or pieces of paper and sticky tape). Each sticker has at least one corresponding equivalent, e.g. 2.3 km, 2300 m. |

Problem Solving

Below are some possible activities linking the problem-solving strand to the knowledge and understanding learning objectives for the unit. To enable effective development of problem-solving skills, you should aim to include problem-solving opportunities in as many lessons as possible across the unit.

| **Activity ideas** | **Resources** |
| --- | --- |
| Learners write down four letters on their whiteboards with their values, e.g. *a* = 2, *b* = 3, *c* = 5, *d* = 10.They explore how many expressions they can find with a value of 20 (e.g. 2*c* + *d*).  |  |
| Challenge learners to find the length of a side of an equilateral triangle (*t*) given that:* the side length of the triangle is 5 cm longer than the side length of a square (*s*)
* the perimeter of the triangle is 8 cm more than the perimeter of the square.
 |  |
| Learners use algebra to solve problems such as:* I think of a number, add 3.7, then multiply by 5. The answer is 22.5. What is the number?
* The sum of two numbers is 12.

The difference of the same two numbers is -4. What are the numbers?* A grocer sells a total of 50 items of fruit in one day. She sells 8 more apples than oranges; how many of each does she sell?
* There are 50 learners in a class; the number of females is 2 more than 5 times the number of males. How many of each in the class?
 |  |
| Learners work in pairs. Each learner secretly draws a function machine labelled with the function it represents. They write down one of the inputs and its matching output for their partner, who tries to work out what the secret function is. They gradually reveal more inputs and outputs if clues are needed. |  |
| Give learners a table of values and several graphs. Learners discuss which of the graphs represents the values in the table, giving their reasons. For example:

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| ***x*** | -4 | -2 | 2 | 0 |
| ***y*** | 8 | 4 | -4 | 0 |

bac155-5-15d-12 -10 -8 -6 -4 -2 4 6 8 10 12 |  |
| Learners draw compound shapes that can be split into rectangles for their partner to find the area and perimeter of*. What is the minimum information you need to give? Have you provided enough information? Have you provided extra information? How did you work out the answer?* |  |
| Learners imagine they have 36 one-metre lengths of fencing. They determine the largest rectangular area of land they could fence off using all of the fencing. Showing their working.Learners now determine the largest area they could fence off if they could use a wall as one boundary, e.g.wallLearners now imagine they can attach the fence to the wall shown above. wallWhat is the largest area that can be fenced off now? Would your answers still be the same if you could use 40 m of fencing flexibly, rather than fixed 1 m lengths? |  |
| Ask learners to solve word problems involving length, mass and capacity, including problems with more than one step. For example:* I am putting up two shelves. Each shelf needs to fit a space that is 87 cm long. I buy a 2 m length of wood. What length of wood will be left over?
* I am following a cake recipe that uses 225 g of flour. I am baking 3 cakes. If I have a 1 kg bag of flour how much will I have left after making the cakes?

Ask: *How did you decide how to solve the problem? How else could you solve the problem? Was your strategy the most efficient?* |  |
| The table shows the distance (in km) between five towns. Five friends live in these five towns and want to meet.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Bergerac |   |   |   |   |
| 87 | Morse |   |   |   |
| 79 | 47 | Langdon |   |   |
| 61 | 31 | 54 | Liberty |   |
| 58 | 84 | 37 | 65 | Marmalade |

Learners use the information to decide in which of these towns the friends should meet to keep the total travelling distance as small as possible. |  |

Unit 2C: Handling Data, Geometry and Problem Solving

Handling Data and Geometry

| Curriculum framework codes | Learning objectives | Activity ideas | Resources |
| --- | --- | --- | --- |
| 7Gs5 | Start to recognise the angular connections between parallel lines, perpendicular lines and transversals. | * Recap the terms 'parallel' and 'perpendicular' by asking learners to work in groups to write a definition of each term. When complete, each group reads out their definition. Introduce the term 'transversal' (a line that cuts across two or more, normally parallel, lines). Assess learners' understanding by asking them to sketch a perpendicular transversal of two parallel lines.
* Learners use non-perpendicular transversals of two parallel lines. They identify equal angles and colour them, within diagrams, through these different tasks:
* drawing and rotating diagrams using a computer program or tracing paper
* colouring equal angles on diagrams with transversals by comparing their sizes with tracing paper
* checking the size of angles in diagrams with transversals using a protractor.

 *What do you notice about angles that are the same? Can you see a pattern? Why do you think these angles are equal?*  *What would be different about the equal angles if the transversal was perpendicular?* (all the angles would be equal – they would all be right angles)  | Tracing paper / acetate sheet for rotatable diagrams or computers with program to draw and rotate diagramsRulersTracing paperProtractors |
| 7Gs6 | Calculate the sum of angles at a point, on a straight line and in a triangle, and prove that vertically opposite angles are equal; derive and use the property that the angle sum of a quadrilateral is 360°. | * *What do you know about the sum of the angles in a triangle?* (The sum is always 180°.)

Demonstrate by drawing a triangle and marking its angles. Then tear it into three parts, each containing an angle, and stick the three angles together, showing a straight line.  In pairs, learners investigate the following statements:  *Is it true that any quadrilateral can be divided into two triangles?* *What is the sum of the angles of any quadrilateral? Why?** Learners remind themselves about relationships between angles. They construct angles around a point, angles on a straight line, vertically opposite angles, triangles, and quadrilaterals. They measure the angles comparing their results with a partner.
 | Large paper triangleRulersProtractors(You could use a dynamic geometry program to construct and manipulate diagrams instead.) |
| 7Gs9 | Recognise line and rotation symmetry in 2D shapes and patterns; draw lines of symmetry and complete patterns with two lines of symmetry; identify the order of rotational symmetry. | * Review line symmetry. Provide learners with 2D shapes that are already drawn and ask them to identify the lines of symmetry.
* Provide learners with 2D shapes that are part drawn (one quarter) and ask them to complete them so that they have two lines of symmetry.
* Review rotational symmetry. Demonstrate the effect of rotating a rectangle around its central point. Draw around a card rectangle on the board and then rotate the rectangle around its centre. Explain that because the rectangle fits on its outline twice in a whole (360°) turn, we say that the shape has 'rotational symmetry of order 2'.

Explain that if a shape will only fit on its outline once, it has no rotational symmetry but this is described as 'rotational symmetry of order 1'. * Remind learners about rotating shapes around one vertex. Ask them to rotate a rectangle 90° around one vertex four times to make a pattern. *What order of rotational symmetry does the pattern have?* (Order 4.) They then rotate the rectangle 45° around one vertex eight times. *What order of rotational symmetry does the pattern have?* (Order 8.)
* Recap that the centre of a rotation can be at any point. Show learners pre-prepared examples.
* Provide learners with patterns that are already drawn and ask them to identify the lines of symmetry and the order of rotational symmetry.
 | Pre-drawn shapes or pre-generated computer shapesPartially complete pre-drawn shapes or pre-generated computer shapesLarge card rectangleSmall card rectanglesProtractorsExamples of rotation patterns showing the centre of rotation in different positions, such as Sheets of pre-drawn patterns, or students can generate computer patterns that they can swap and compare with each other |
| 7Dp1 | Find the mode (or modal class for grouped data), median and range. | * Recap how to find the mode, median and range. Provide data sets and ask them to identify the data set for which e.g. the mode is 7 and the range is 6.
* In groups, learners compare the temperatures of possible holiday destinations by analysing the range, mode and median. *Why is it useful to know the range … the mode … of the temperatures? Which destination would you recommend I go to based on all of the statistics? Why?*
* Learners find the median, mode and range of different sets of data. *What does the median … mode … range tell you in the context of this data?*
* Display a frequency table showing grouped data. *How can you find the mode of grouped data?* Establish that you are finding a modal class rather than a single piece of data.
 | A selection of data setsSimple frequency tables showing, e.g. July temperatures from various holiday destinationsFrequency table showing grouped data |
| 7Dp2 | Calculate the mean including from a simple frequency table. | * Explain that you are thinking of taking a holiday in July but you are not sure where to go. You would like to go somewhere warm, but not too hot. You have several possible destinations and you want learners’ help to choose.

Give each group a frequency table showing 31 days of temperatures from the previous July in a different destination. Elicit from the learners how they could analyse the data. Recap on the definition of ‘mean’ and how to calculate it. Learners find the mean of their destination’s temperatures.*What does your mean tell you?* Groups share their means. *Which do you think would be the best holiday destination? Why?** Learners roll dice to create a frequency table and find the mean of the numbers rolled.
* Learners work in pairs to measure each other’s heights to form a set of class data. Learners then find the mean of all the heights. Calculators can be used to help with the calculation. *What does the mean tell you about the heights? What does it NOT tell you?*
 | Simple frequency tables showing July temperatures from various holiday destinationsDiceMeasuring tape or metre rulersCalculators |
| 7Dp3 | Draw and interpret:* bar line graphs and bar charts
* frequency diagrams for grouped discrete data
* simple pie charts
* pictograms.
 | The activities below are in the context of bar line graphs and bar charts. They can be adapted for frequency diagrams for grouped discrete data, simple pie charts and pictograms:* Show learners a range of bar-line chart and bar charts. *What can bar-line charts and bar charts be used to represent? What is similar … different about the two kinds of charts?* Put together a reminder list of the conventions related to bar-line charts and bar charts such as title, labels on axes and an appropriate scale.
* Provide pairs of learners with a range of bar-line charts and bar charts that do not follow convention. Ask learners to critique the charts and say how they could be improved.
* Provide learners with a set of data and ask them to draw a bar chart or bar-line graph. Learners then interpret their diagram (Note: drawing graphs and charts using software speeds up the process and leaves time for learners to interpret the chart, which is an important focus.) Interpretation questions could include: *Which is the most common/popular …? What is the difference between the most and least common/popular …? Why do you think that there is a difference between …?*
* Collect a range of different representations of data from the media. Discuss with the class what each chart shows and the extent to which the chart is appropriate, informative, misleading etc*.*
 | Range of examples of bar-line charts and bar charts that follow convention (You could undertake an internet search to look for examples. For this activity use charts that are correctly set out and show the correct conventions. Save incorrect examples for the second task.)Range of examples of bar-line charts and bar charts that **do not** follow convention. You could use previous learners’ examples, charts that you have created or incorrect examples from the internet.Pre-prepared set of dataA range of representations of data in the media |
| 7Db4 | Identify all the possible mutually exclusive outcomes of a single event. | * *What are all the possible outcomes when throwing one die?* List all the possibleoutcomes.

*What is the probability of throwing a 1 on a 6-sided die? Why? How does knowing all the possible outcomes help you to find the probability?*Learners list some of their own situations and probabilities when throwing a 6-sided die, e.g. throwing* an even number (1/2)
* a number greater than 4 (2/6 = 1/3)
* a multiple of 3 (2/6 = 1/3)

They share their situations with a partner, who works out the probabilities for themselves. *How did you work out that probability? How can you express that as a percentage?* * Learners carry out the activity above for dice with different numbers of sides.
* In pairs, learners set each other probability questions about drawing a playing card from a full pack of cards. For example:
* What is the probability of drawing an 8?

What is the probability of drawing a heart?  | 6-sided diceDice with different numbers of sides (or spinners)Pack of cards |
| 7Db57Db6 | Use experimental data to estimate probabilities.Compare experimental and theoretical probabilities in simple contexts. | * Establish the difference between theoretical probability and experimental data.

Referring back to earlier work, ask learners to predict the probability of throwing a 6 on a 6-sided die. Learners then work in pairs to carry out an experiment for 60 throws, recording the outcomes on a tally chart. *How did you make your prediction? Does your experiment support your prediction?* Learners analyse the results for each pair and then put the data from other pairs together to produce more reliable results.*Why are some individuals’ outcomes quite different to the outcomes of others? Why don’t we toss exactly 1/6 sixes and 5/6 non-sixes?* * Working in groups, learners are given a spinner or a bag of differently coloured objects. *What is the theoretical probability of getting each number/colour? Why?*

 They spin/pick 50 times (putting the object back in the bag before the next turn) and record the outcomes.They find the total of each outcome and record their results. Groups then circulate to another table to analyse another group's results. They try to decide what the spinner numbers / object colours must be to match the results. *How did you decide? How does your prediction match the actual spinner/bag? Was the experimental data misleading in any way? What is the theoretical probability … experimental probability?*  | One 6-sided dice per pairOne spinner or bag of coloured objects per group (different for each group):Spinners – six-sided spinners showing six numbers, some of which are the same, e.g. 1, 1, 1, 2, 3, 4 or 1, 2, 3, 3, 4, 5 or 2, 2, 3, 3, 4, 4Coloured objects in an opaque bag, e.g. 2 blue, 4 green, 1 white, 1 yellowor 3 green, 3 white, 3 yellow, 1 purple  |

Problem Solving

Below are some possible activities linking the problem-solving strand to the knowledge and understanding learning objectives for the unit. To enable effective development of problem-solving skills, you should aim to include problem-solving opportunities in as many lessons as possible across the unit.

| **Activity ideas** | **Resources** |
| --- | --- |
| This is a not-to-scale diagram of a roof truss. 17°Learners work in pairs to calculate all the possible angles. They also identify any angles they cannot calculate, giving reasons. Learners say what further information they require to be able to calculate these angles. |  |
| Learners explore the problem below, trying to find all the possible solutions. They have to explain how they are sure they’ve found them all.The three pieces below can be fitted together to make shapes with at least one line of symmetry.Each piece must touch another edge-to-edge, and two pieces must not overlap.Learners could design their own set of three shapes with a total area of 10 square units and challenge a partner to find all the ways they can they be arranged to make symmetrical shapes.Can you produce a set of three shapes with a total area of 10 square units which can be arranged into more symmetrical shapes than those in the original problem? | Square grids, card for drawing shapes |
| Learners determine what is the smallest number of additional squares that must be shaded so that this figure has at least one line of symmetry *and* rotational symmetry of order 2. |  |
| Sami can catch either Bus A or Bus B on his way home from work. The table below shows the waiting times in minutes for the last five journeys on each bus.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **Bus A** | 10 | 8 | 5 | 9 | 8 |
| **Bus B** | 16 | 1 | 2 | 15 | 1 |

Learners predict which bus it would be more sensible to catch, by finding the mean waiting time. They explain their decision. *What other information would Sami need to make a final decision?* (e.g. mean journey time, time to walk to/from the bus stop) |  |
| Learners use their knowledge of statistics to solve problems such as:* Shami’s class take five mathematics tests across the year.

His scores on his first four tests were 80, 85, 88 and 95.To get a grade A, he needs to have a mean of 90 or better. What score must he get on the fifth test to get a Grade A?* Raina writes down six numbers, four of which are 10.

The numbers have a mean of 10 and a range of 6. What are the other two numbers?* Can you find sets of five positive single-digit integers where the mode, median, mean and range are all the same? (You could model this problem first using the set 1, 2, 2, 2, 3 as an example.)
 |  |
| Ask learners to give a short talk on the environment. They must choose two issues using the information given on the bar chart. They have to explain why they chose each issue using only the information in the chart. Do Now Would think  about doingUse ozone friendly aerosolsPick up other people’s litterTake bottles to bottle bankCollect newspapersUse recycled paperEat organic foodRecycle kitchen wastePercentage 0 20 40 60 80 100 |  |
| Show a bar chart showing the following rainfall data in mm for four months:

|  |  |
| --- | --- |
| June | 16 mm |
| July | 10 mm |
| August | 12 mm |
| September | 14 mm |

Draw a horizontal line on the bar chart at 9 mm, and state that this represents the mean rainfall for the four months.Learners discuss whether this is true or not, giving reasons for their decision. |  |
| There are 6 balls in a bag. The probability of taking a red ball out of the bag is 0.5. A red ball is taken out of the bag and not replaced. *What is the probability of taking another red ball from the bag?* *How many balls need to be removed without replacing, before the probability of getting a red ball is 0? Is this always the case?*  |  |

Unit 3A: Number, Calculation and Problem Solving

Number and Calculation

| Curriculum framework codes | Learning objectives | Activity ideas | Resources |
| --- | --- | --- | --- |
| 7Nc8 | Add and subtract integers and decimals, including numbers with different numbers of decimal places. | * Recap how to set out written additions and subtractions using columns. Emphasise the importance of using place value.
* Give each learner a set of 0–9 digit cards. Learners shuffle their cards and then position the 10 digits in two rows of five to form a written calculation. They add and subtract the two numbers using written methods. *What do you need to make sure of when you position your cards?* (Digits with the same place value in the same column; the bigger number on top for the subtraction.)

Challenge learners to be the fastest to give the correct answer to given written additions and subtractions. Alternatively, time how long it takes learners to complete 10 given calculations. Then repeat with 10 more calculations, challenging them to improve their time.Learners work in pairs, each with their own calculation and their own set of cards. Each learner makes two decimal numbers by turning one-digit card over at a time and placing it in a grid like this: **.****.****.** After the digits have been placed, learners then decide where to position the decimal point. *Who can make the smallest total? Who can make the smallest difference? Who can make the largest total/difference? Who can make an even/odd total/difference? A total/difference that is a multiple of five? What difference would there have been if the decimal point was in a different place? What effect would it have had on your answer – would you have still won/lost that round? Why?* | A set of 0–9 digit cards per learnerThree decimal point cards per learner |
| 7Nc9 | Multiply and divide decimals with one and/or two places by single-digit numbers, e.g. 13.7 × 8, 4.35 ÷ 5. | * Recap how to set out written multiplications and divisions using columns. Emphasise the importance of using place value.
* Learners work in pairs. Learner A works out the answer to 137 × 8 and Learner B works out the answer to 13.7 × 8. They then compare their answers.

Discuss the similarities and differences in the answers as a class. Relate back to the importance of place value in written multiplications. * Use the activity above for the divisions 435 ÷ 5 and 4.35 ÷ 5.

Learners have two sets of cards. They take a decimal from Set 1 and single-digit number from Set 2. They estimate the product, and then calculate the answer. They discuss their strategies with a partner. *How did you find your estimate? How close was your estimate to the answer? Do you think your estimate was appropriate? Why? Did you choose to calculate mentally or using a written method? Why?** Learners create decimal divisions for a partner to answer. When creating their divisions, they must know the answer themselves in order to check their partner’s answers.

*Did you choose to calculate mentally or using a written method? Why?* | Two sets of cards per learner:Set 1: 8 cards showing numbers with one or two decimal placesSet 2: 2–9 digit cards |
| 7Nc10 | Know that in any division where the dividend is not a multiple of the divisor, there will be a remainder, e.g. 157 ÷ 25 = 6 remainder 7. The remainder can be expressed as a fraction of the divisor, e.g. 157 ÷ 25 = 6 7/25.  | * In pairs, learners review the idea of division with a remainder by dividing sets of beads or counters into groups. They write each division as a number sentence with an integer remainder, e.g. 19 ÷ 5 = 3 remainder 4

*When is there a remainder?* (when the dividend is not a multiple of the divisor).With learners’ help, model a written method for division. *What is the remainder as a fraction? Why*? * In pairs, learners use digit cards to generate divisions of the form:

 🞏🞏🞏 ÷ 🞏 They calculate the answer individually, expressing any remainder as a fraction. They then compare their answer with a partner’s. *Will there be a remainder? Why / Why not? How did you decide how to express the remainder as a fraction?* | Beads or counters (or stones)Sets of digit cards |
| 7Nc11 | Know when to round up or down after division, when the context requires a whole-number answer. | * Learners discuss division problems which involve decisions about whether to round up or down, and why, e.g.
* 124 learners are going on a school trip. Each bus has 40 seats. How many buses are will be needed?
* Eggs are packed in boxes of 9. How many boxes do 186 eggs require?

Discuss learners’ strategies as a class.* Provide learners with word problems with gaps for learners to insert their own decimals and single-digit numbers before solving, e.g.

Sonja has a bottle of medicine. It contains \_\_\_\_\_ litre. She uses one quarter of the medicine. If a spoonful holds \_\_ ml how many spoonfuls are left? *Does your answer make sense in the context?* |  |
| 7Nf6 | Calculate simple percentages of quantities (whole number answers) and express a smaller quantity as a fraction or percentage of a larger one. | * Revisit calculating simple percentages of quantities.Ask learners to say which of a pair of given percentages of quantities is bigger, e.g.

**A** 20% of 50 kg or **B** 15% of 70 kg.*How did you decide?** Ask the class to arrange themselves into two groups according to given criteria, e.g. girls in one group, boys in the other. Ask questions about the fraction, and then the percentage, of the class in each group:

*What fraction of the class are boys? … girls?**How can we find out what percentage of the class are boys? … girls?**How can we check our answers?* (e.g. The fractions should add to 1. The percentages should add to 100%.)Repeat for other groupings, including where there are more than two groups.  |  |
| 7Nf7 | Use percentages to represent and compare different quantities. | * Using a collection of packaging showing statements such as ‘50% extra free’, learners record the original and revised quantities. *What percentage of the old size is the new size? How can you work out a new size from the original size?*
* Using items with ‘sale tags’, learners decide which item is the best deal. For example:

 *Should I buy Item 1 costing $450 with 35% off, or Item 2 costing $400 with 25% off?** Show two different food containers. Ask, e.g. *Which would you prefer – the 50% of the ice cream that is left in this container, or the 40% that is left in this container? Why?* Use popular foods and unpopular foods.
* Provide learners with problems comparing two different school populations, e.g.

 48% of the learners at School A are girls. There are 600 pupils in School A. In School B there are 650 pupils and 45% are girls. Which school has the most girls? *Why are percentages a useful way to compare two sets that are not the same size?* | Collection of packaging/items that show statements such as ‘25% extra free’ (e.g. cereal boxes, bottles)Items labelled with mock prices and sale tags (or items on the internet)2 different sized containerslabelled with their masses |
| 7Nf8 | Use ratio notation, simplify ratios and divide a quantity into two parts in a given ratio. | * Explain the term 'ratio' using pictures or objects to exemplify:
* Ratio compares part with part, e.g.
* ‘for every…, we need …’ The relationship between these numbers is expressed in the form ‘a to b’ or more commonly in the form ‘a : b’.
* Learners write the ratio notation that represents given pictures, e.g. the ratio of red counters to blue counters. Learners show their answers on mini whiteboards. Establish that the order of the ratio is important, e.g.

 ratio of red to blue = 2 : 3 is different from ratio of red to blue = 3 : 2 but is equivalent to ratio of blue to red = 3 : 2.* Provide learners with a set of buttons, or similar. Learners use the objects to show different ratios.

Ask them to identify and justify equivalent ratios, e.g. 1 : 2 = 2 : 4. *What does your ratio mean?* (e.g. 1 red button to every 2 blue buttons.) *So, what ratio is equivalent to 1 : 2? Why?* (e.g. 2 : 4 because this still gives 1 red to every blue.) * Once learners have had practical opportunities to divide quantities into two groups in a given ratio, show the learners how to use a bar model. For example, divide 24 in the ratio 5 : 3:

*bar%20model*Give learners other quantities to divide in different ratios. Encourage learners to explain their reasoning. Have coloured objects available for learners to use to check their answers.* Give learners ratios (including in context) to simplify, e.g.
* 5 : 10… 6 : 15 … 132 : 60
* The number of boys in a club is 12 and the number of girls is 16. Express the ratio of boys to girls in its simplest form.
* The park has 2800 square metres of flower beds and 4400 square metres of grass. Express the ratio of flower beds to grass in its simplest form.

 *How do you know that is the simplest form?* | Pictures or objects in two different coloursMini whiteboards and pensSets of coloured objects, e.g. buttons or painted stonesPre-prepared ratios to simplify, including word problems |
| 7Nf9 | Recognise the relationship between ratio and proportion. | * Clarify the definition of ‘proportion’ with learners.
* Proportion compares part with whole, e.g. … out of …
* It is usually expressed as a fraction, decimal or percentage.

Clarify the difference between ratio and proportion by asking learners to express ratios and fractions for sets of pictures or objects.* Discuss where proportion and ratio are used in everyday life, e.g.
* proportion: eating ¼ of a pie, running 2/3 of a race
* ratio: converting kilometres to litres, sizing recipes up/down
* Give learners a set of proportion statements. They first express a statement using fractions, decimals and percentages to practise using the vocabulary of proportion interchangeably, e.g*.*

 In one litre of squash, 200 ml is concentrate: can be written as  1/5 of the squash is concentrate.* 1. of the squash is concentrate.

 and  20% of the squash is concentrate. Encourage learners to try to write an equivalent ratio statement, e.g. The ratio of concentrate to water is 1 : 4. Ask learners to write their fractions and ratios in their simplest form, as in the examples above. *What do you notice about the patterns and relationships between ratios and proportions*? * Learners work in small groups to complete a partially completed table, such as the one shown on the right. Encourage discussion about relationships, simplifying and possible contexts that match the proportion statements.

Establish that a ratio *a* : *b* is equivalent to the proportion *a*/a *+ b*. | Pictures or objects in two different colours (for learners to express as fractions and ratios)Proportion statement cardsLarge worksheets showing partially completed tables, e.g.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Proportion statement | Ratio | Fraction | Percentage | Decimal |
| 20 out of 100 | 1 : 4 | 1/5 | 20% | 0.2 |
| 35 out of 70 |  |  |  |  |
|  |  |  | 75% |  |
|  | 2 : 7 |  |  |  |
|  |  | 2/3 |  |  |

 |
| 7Nf10 | Use direct proportion in context; solve simple problems involving ratio and direct proportion. | * Explain the meaning of ‘direct proportion’: If two amounts are in direct proportion, as one amount increases, the other amount increases at the same rate.

Discuss the example of hourly pay. How much you earn is directly proportional to the number of hours you work. The more hours you work, the more you get paid, in direct proportion.  When your pay is $5 an hour: If you work 1 hour you get $5  If you work 2 hours you get $10  If you work 3 hours you get $15  and so on. Pay is $ = 5 × hours worked. We call 5 the ‘multiplier’.* Encourage learners to think about multipliers by carrying out a ‘broken calculator’ activity. In pairs, learners use a calculator to change one number into another. They are only allowed to use the × and ÷ buttons (because the addition and subtraction buttons are broken), e.g.

 Change 10 into 15 … 8 into 20 … 5 into 8 … 1.5 into 8.5 … 1.2 into 3.4. *What is the multiplier?** Provide learners with contexts where they are required to use direct proportion in context, e.g.

 In a recipe for 4 pancakes you need:  6 dessertspoons flour ¼ litre milk 1 pinch salt 1 egg*How many eggs are needed to make 8 pancakes? How much milk is needed to make 10 pancakes? How do you know? What multiplier did you use? Are your answers reasonable? Why?** Provide learners with a shopping list. Learners compare the prices of different sized packets and work out which is the best value. *How did you work out which of these two items is the better value?*
 | CalculatorsPrepared direct proportion contexts e.g. simple recipesShopping listsAccess to a supermarket website or food items / empty packages labelled with prices  |

Problem Solving

Below are some possible activities linking the problem-solving strand to the knowledge and understanding learning objectives for the unit. To enable effective development of problem-solving skills, you should aim to include problem-solving opportunities in as many lessons as possible across the unit.

| **Activity ideas** | **Resources** |
| --- | --- |
| Give learners three numbers, e.g. 2, 3, 4. Learners work in pairs to make as many different answers as they can, using brackets, squaring, addition, subtraction, multiplication and division. Each number can be used a maximum of two times. Not all numbers need to be used each time.  |  |
| A woman goes into a supermarket and buys four items. She decides to use her calculator to find the total, but she multiplies the costs (in dollars, using the decimal point for the cents) instead of adding them.At the checkout she says, "So that's $7.11." The checkout man, who has correctly added the items, agrees.Learners find four possible prices of the items, deciding whether there is only one answer or more than one answer. |  |
| Learners compare prices in two shops to find the better buy. Both shops have discounts, e.g.*Jones* is selling:* trainers originally $40 but with 10% off
* tops originally $40 but with 50% off
* caps originally $10 but with 20% off.

*Gills* is offering the same items but:* trainers originally $30 but with 1/3 off
* tops originally $11 but now ½ price
* caps originally $44 but with ¼ off.

Learners investigate which shop is cheaper for1. trainers
2. a top
3. a cap
4. a cap, top **and** trainers.
 |  |
| Some lemonade was mixed up in two glasses.The first glass had 200 ml of lemon juice and 300 ml of water.The second glass had 100 ml of lemon juice and 200 ml of water.Learners decide, giving mathematical reasons, which mixture has the stronger tasting lemonade.They then compare different mixtures of lemonade and develop a strategy for deciding which is stronger each time. *How might you use fractions to help you to work out which mixture is stronger?*Learners then explore different combinations, e.g. if the two glasses of lemonade (above) are mixed together, the new mixture is weaker than the first glass was, but stronger than the second glass. *Is the strength of the combined mixture always between the strengths of the originals?* Learners discuss and communicate their findings, providing justifications and reasons. |  |
| Pose problems such as:* Two learners share $*x* in the ratio 5 : 7. The smaller share is $15; how much is the larger share?

 *How did you work it out?*  |  |
| The points A, B, C and D all lie on the same straight line. The distance from A to D is 164 cm. The ratio of the length AB to BC is 2 : 5. The ratio of the length of BC : CD is 3 : 4. Find the distance from B to C. |  |
| * Provide a range of word problems involving direct proportion, such as those below. Learners solve them individually and then compare their answers with a partner’s, discussing any discrepancies.
* Yana travelled at 30 km/h for 30 minutes. If she travelled at 60 km/h over the same distance, how long would it take?
* It takes 10 hours to fill a 200 m3 tank using four pipes. How long would it take to fill a 500 m3 tank using two pipes?

*What multiplier did you use? How did you work it out? Does your answer make sense in the context of the problem? Why?* | Pre-prepared word problems involving direct proportion |

Unit 3B: Measure and Problem Solving

Measure

|  |  |  |  |
| --- | --- | --- | --- |
| **Curriculum framework codes** | **Learning objectives** | **Activity ideas** | **Resources** |
| 7Mt2 | Know the relationships between units of time; understand and use the 12-hour and 24-hour clock systems; interpret timetables; calculate time intervals. | * Set a time on a clock. *What time would it be in … minutes?* (e.g. 45 minutes, 97 minutes). Repeat several times, including some times in the past too – *What time was it … minutes ago?* Use both analogue and digital clock times. You could invite learners to pose some questions for the class too.
* *What is the advantage of using 24-hour clock times?* Clarify that 12-hour times need to include indication of whether they refer to morning (between 12 midnight and 12 noon) and afternoon/evening (between 12 noon and 12 midnight). 24-hour clock times do not, so they can be clearer. *When might we use 12-hour … 24-hour clock times?* (e.g. 12-hour clock times when we’re talking about the time now, but 24-hour clock times on travel timetables)
* Ask questions relating to the units of time. *How many seconds are there in a minute? … minutes in an hour? … hours in a day?*

* Revise converting between 12-hour and 24-hour clock times. Learners play a matching card game in pairs or small groups. They arrange the cards face down on the table. They take turns to turn over two cards. If the two cards match, they keep them and take another turn. If the two cards don’t match, play passes to the next player. The winner is the learner who collects most cards.
* Ask learners to work in pairs. Give each pair two bus timetables for routes that have at least one stop in common. Ask them to work out which two buses they would need to catch to get from A to B by 11am. *Is there only one possibility?*
* Give each learner a mini whiteboard and a pen. Show them two clocks with different times (e.g. 10:10 and 11:40). Ask them to write the difference in time between the two. *What happens if one clock is showing am and one is showing pm?*
 | Digital and analogue clocksSets of cards showing matching 12-hour and 24-hour times on different cardsCopies of two bus timetables for routes that have at least one stop in commonMini whiteboards and pens (one each per learner) and two analogue clocks |
| 7Ma3 | Derive and use the formula for the volume of a cuboid; calculate volumes of cuboids. | * Define ‘volume’ as the amount of 3D space a shape occupies.

 Explain that units of volume are written with 3. Show 1 cm3 cubes to distinguish between cm2 and cm3. *What is special about a 1 cm3 cube?* (Its length, width and height all measure 1 cm.)* With learners' help, make a cubic metre out of 12 metre rulers and compare it to a 1 cm3 cube. *How many 1 cm3 cubes would be required to make 1 m3? Why?*
* In groups, learners build a range of different cuboids using 1 cm3 cubes. *Can you find a general rule … formula for calculating the volume of cuboids?* Discuss findings. Establish that the volume of a cuboid is the area of the base (length ×width) multiplied by the height:

 Volume= length × width × height *V* = *lwh** Learners roll a die three times to generate three centimetre lengths. They sketch a cuboid with those dimensions and calculate the volume of the cuboid. They share their volume with a partner, who tries to work out the dimensions of the cuboid.
* Learners calculate the volumes of cuboid boxes from the nets. *Which measurements do you need?* *Which parts of the net will you measure?* Learners check their volumes by folding the nets into cuboids and measuring and calculating again.
 | 1 cm3 cubes12 metre rulers (or 1 m sticks) and tape or string to hold them together1 cm3 cubes DiceA variety of empty cardboard boxes (cuboid or cube) opened out into netsRulersCalculators (if required for calculations) |
| 7Ma4 | Calculate the surface area of cubes and cuboids from their nets. | * Provide learners with cuboid boxes that are opened out into nets. Ask learners to calculate the surface area from the nets. *What is your strategy? Do you need to measure every edge? Do you need to calculate the area of every side?*

Establish that the opposite sides of a cuboid are the same, so you can find the sum of the areas of the three different sides and then multiply by 2. *How can you find the surface area of a cube?*  | A variety of empty cardboard boxes (cuboid or cube) opened out into netsRulersCalculators (if required for calculations) |
| 7Mt1 | Draw and interpret graphs in real life contexts involving more than one stage, e.g. travel graphs. | * Invite a learner to provide data for a distance–time graph. They walk slowly in one direction and then stop for a little while before moving back to the starting point a little more quickly. The class collects time and distance information from the starting point. They graph it in real time, ideally in a spreadsheet to see the graph change every few seconds.

 Discuss how the graph relates to the movement*. At which point on the graph does … start moving back? Is the graph symmetrical? Why / Why not? Does the graph represent discrete or continuous data? Why?* (Continuous data because all points on the graph – even between marked times – have meaning.)* Learners plot distance–time graphs for their journey from home to school, or other one-way distance–time graphs in familiar contexts. A partner interprets the graph by describing the journey. *Why doesn't the graph go back to 0 km?* (Because it's a one-way journey.) *Did you travel at the same speed all the time? How does your graph reflect this?*
* Provide learners with a distance–time graph for a journey. They describe the journey to a partner. They discuss any differences in their descriptions. *What does this part of the graph show? How do you know?*
* Provide learners with graphs representing how a bath fills with water over time. Ask them to describe what is happening in each graph.
 | Open spaceSpreadsheet with graphing function (or graph paper and rulers)Tape measureTimerSquared paperRulersReady-plotted distance time graphsReady-plotted graphs representing how a bath fills with water over time, e.g.Screen%20Shot%202018-01-08%20at%2013 |

Problem Solving

Below are some possible activities linking the problem-solving strand to the knowledge and understanding learning objectives for the unit. To enable effective development of problem-solving skills, you should aim to include problem-solving opportunities in as many lessons as possible across the unit.

| **Activity ideas** | **Resources** |
| --- | --- |
| Sometime during every hour, the minute hand lies directly above the hour hand.clock imageAt what time between 4 and 5 o'clock does this happen? When else does this happen? |  |
| Given a fixed number of 1cm3 cubes (i.e. a fixed volume), learners find the cuboid with the largest surface they can build. They repeat using different numbers of cubes. | 1cm3 cubes |
| Learners work in groups. Challenge them to make as many different cuboids (or cubes) as possible with a given surface area, e.g. 72 cm2. Give each group a different surface area. They make their cuboid by first drawing its net on squared paper. *What strategies are you using? Have you found all of the possible cuboids with integer side lengths? How do you know?* *What are the volumes of your cuboids? Which has the largest volume?*Extend the challenge by asking one group to give the surface area and volume of one of their cuboids to another group. The other group has to try to make the exact cuboid. | Squared paperSticky tapeScissors  |
| Ask learners to imagine a 10 × 10 grid of squares.If each corner square is cut out, the paper can be folded to make a tray or open-topped box with a base of 8 squares by 8 squares and it would be 1 square deep. The tray would be able to hold 64 cubes of side 1 square each.*What would happen if you cut out 4 squares at each corner?**What would happen if you cut out 9 squares at each corner?**Which gives the maximum volume?* |  |
| A playground is rectangular and its surface is covered in rubber for safety.The length of the rectangle is 20 metres less than twice the width. The thickness of the rubber is 1.5 cm.Learners work in pairs to formulate an expression for the volume of rubber *V* used to cover the playground, in terms of the playground’s width *w*. Once learners have the expression, they use it to discover how much rubber there is if the width is 90 metres.They challenge each other to work out the volume of rubber for different-sized playgrounds, giving either the length or the width. |  |
| Give learners a distance–time word problem and ask them to graph the problem in order to find the solution, e.g. A group of 10 learners are on a field trip when their bus breaks down 40 km away from school. A teacher takes 5 of them back to school in her car, travelling at an average speed of 40 km per hour.The other 5 learners start walking towards school at a steady 4 km per hour. The teacher drops the first 5 learners at school, then immediately turns around and goes back for the others, again travelling at a steady speed of 40 km per hour. How far have the learners walked by the time the car reaches them?*How will your graph reflect the teacher's journey and the walking learners? Where will your graph start? Why? What is the key information for the next part of your graph?* | Distance–time problemsGraph paper / squared paperRulers |

Unit 3C: Handling Data, Geometry and Problem Solving

Handling Data and Geometry

| Curriculum framework codes | Learning objectives | Activity ideas | Resources |
| --- | --- | --- | --- |
| 7Dc3 | Construct and use frequency tables to gather discrete data, grouped where appropriate in equal class intervals. | * Give learners topics for data collection that will result in discrete data that can be grouped. After collecting their data, learners refine their ideas about how they will organise and present their data. *Will you group your data? Why? What class intervals will you use? Why?* Remind learners that when data values are spread out, it is difficult to set up a frequency table for every data value, as there will be too many rows in the table*.* You can group the data into equal class intervals to help you to organise, interpret and analyse the data*.*
* Provide learners with a list of data, such as the heights of each of the learners in the class. Ask them to represent the data in a frequency table. *What class intervals will you use? Why?*
 |  |
| 7Di2 | Compare two simple distributions using the range and the mode, median or mean. | * Learners find the mean, median, mode and range of different sets of data. *What does the mean … median … mode … range tell you in the context of this data?*
* In pairs, learners compare the mean, median, mode and range for journey durations using travel timetables. They use their findings to choose the best transport option. *How did you decide? Which statistic helped you most? Why?*
* Learners use statistics to analyse data related to environmental issues, e.g. they could compare recent flood or rainfall data with data from 50 years ago. *What does the mean … median … mode … range tell you for this data?*
 | Pre-prepared sets of dataDifferent travel timetables for transport to a destinationData related to environmental issues |
| 7Di1 | Draw conclusions based on the shape of graphs and simple statistics. | * In groups, learners create charts/graphs of the temperatures, rainfall and hours of sunshine in different holiday destinations. *How did you decide which type of chart/graph to use?* They choose a destination based on their graphs and charts and explain their choice.
* Provide groups of learners with graphs and statistics, asking them to analyse them and draw conclusions.
* Give learners the test marks for two classes. In pairs, learners identify questions for which the data may provide answers and then analyse the data to find the answers.
* You could ask learners to apply their learning across Stage 7 by carrying out a group project to:
* decide on a research question
* plan what data to collect
* collect and organise data
* present data and/or process it using statistics
* interpret and discuss results
* reflect on the project, e.g. *Do you think you have collected enough data? How could you collect more data? Are there any other conclusions you could have reached? What if you had used another source for your data? Do you think your findings would have been different if you had carried out your research at a different time? What could you investigate next?*

Once learners have finalised their conclusions about their research question or hypothesis, they then prepare a presentation of their project to the class. *What information will be most interesting for the class? How will you present what you did … how you did it … what you found out? How will you keep the class interested? How will you make sure you are all involved in the presentation?* | Simple frequency tables showing July temperatures, rainfall and hours of sunshine from various holiday destinationsGraphs and statistics for learners to analyse. If possible, link to a current topic in a different subjectTest scores from two classes. (If using real test scores make sure the data is totally anonymous) |
| 7Gp1 | Read and plot coordinates of points determined by geometrical information in all four quadrants.  | * Learners draw a map of the classroom using a four-quadrant coordinate grid. They write the coordinates of five objects in the classroom. They share their coordinates with a partner, who tries to identify the objects.
* Learners draw a coordinate grid on a copy of a map from an atlas and give

coordinates of places on the map. A partner has to identify the places.* Learners plot points and solve problems based on them, e.g.
* Plot (-3, 1) and (2, 1). If these are two vertices of a rectangle, what could the coordinates of the other vertices be?

 Learners set each other similar problems drawing on their knowledge of the properties of quadrilaterals.* In pairs or small groups, learners take turns to select a card and plot the point. After an agreed number of turns, learners join the points and identify the shape they have created.
 | Coordinate grids with four quadrantsCopies of a mapCoordinate grids with four quadrantsRulersCoordinate grids with four quadrantsSets of cards showing coordinatesRulers |
| 7Gp2 | Transform 2D points and shapes by:* reflection in a given line
* rotation about a given point
* translation.

Know that shapes remain congruent after these transformations. | * Ask learners to come up with a definition for congruent*. How mathematically accurate can they be?* Learners may come up with: ‘The same shape and size’ or ‘Two shapes are congruent when you can turn, flip and/or slide one so it fits exactly on the other.’ Display the agreed definition.
* Provide learners with a number of shapes drawn on paper in various orientations. Some of the shapes should be congruent. *Which of the shapes are congruent? How do you know?* If learners are not sure about some shapes, they should cut them out so they can compare them directly.
* Modelling on a large coordinate grid, recap on transformations. Establish that:
* a reflection is defined by specifying a mirror line, e.g. ‘Triangle ABC is reflected along the *y*-axis.’
* rotation is defined by specifying the centre of rotation, the amount of turn and the direction of rotation (clockwise or anticlockwise), e.g. 'Triangle ABC is rotated through 90° in an anticlockwise direction about (0, 0).'
* a translation is defined by specifying the distance and the direction of a movement, e.g. 'Triangle ABC is translated 2 units to the right.'
* Give learners three coordinates. Ask them to join the points and then rotate the triangle 90° anticlockwise around (0, 0) three times. *What do you notice about the coordinates of the new points?* Give learners three new coordinates. *Where would the points of the triangle be when it has been rotated through 90° about (0, 0)? … 180°? …. 270°? …360°?*
* Provide shapes on a coordinate grid and ask learners to draw the shapes and/or write the coordinates after a translation of a given number of units parallel to the *x*-axis or *y*-axis, e.g. -7 units parallel to the *x*-axis. Establish that a negative unit moves to the left or down and a positive unit moves to the right or up.
* Learners work in pairs and create problems using reflection, rotation or translation for their partner to solve. For example, for reflection they might:
* give the coordinates of a shape's vertices and ask their partner to reflect the shape in a given mirror line
* draw a shape and its reflection and ask their partner to identify the mirror line.
* Ask learners to give facts about a shape and its reflected/rotated/translated image, e.g. for reflection:
* the shape and its image are congruent
* the image is a flipped version of the shape
* the shape and its image are the same perpendicular distance from the mirror line but on opposite sides.
* Provide examples of a range of patterns on carpets, flags, etc. In pairs, learners discuss and identify what transformations have been used to create the patterns. *What clues are you using to identify reflections? …rotations? … translations? How many lines of symmetry does the pattern have? What is its order of rotational symmetry?*
 | Pre-drawn shapes on a coordinate gridSheets showing some shapes that are congruent and some that aren't ScissorsLarge coordinate gridFour-quadrant coordinate grids or squared paperRulersProtractors Shapes on a coordinate gridFour-quadrant coordinate grids or squared paperExamples of patterns, e.g. carpets, flags |
| 7Gs7 | Solve simple geometrical problems by using side and angle properties to identify equal lengths or calculate unknown angles, and explain reasoning. | * Learners work in pairs to discuss and calculate the unknown angles in diagrams.

Discuss strategies as a class.* In small groups learners discuss questions about the sides and interior angles of triangles and quadrilaterals, e.g.
* One angle of a right-angled triangle is 34°. What are the other two angles?
* Two sides of a kite are 3 cm and 6 cm. What are the lengths of the other sides?

 *What shape property helped you to answer that question?** Learners make up their own missing angle diagrams or shape property questions on separate pieces of paper. They write the answers on the back of each piece of paper. Randomly distribute learners' missing angle diagrams and shape property questions. In pairs, learners discuss and solve them. *Does your answer match the one on the back of the paper? What knowledge did you use? Is there more than one strategy for this question?*
* In small groups, learners discuss the unknown angles in diagrams of triangles and quadrilaterals that include exterior angles and require two steps to solve, e.g.

*Do you have enough information to calculate this angle? Why / why not? What knowledge can you use to find this angle?* | Pre-prepared missing angle diagrams with one angle missing (angles at a point, on a straight line, in a triangle, in a quadrilateral and vertically opposite angles)Pre-prepared questions about the sides and interior angles of triangles and quadrilaterals |
| 7Gs8 | Recognise and describe common solids and some of their properties, e.g. the number of faces, edges and vertices. | * Recap the definition of polyhedra.Show learners a variety of shapes, asking: *Which of these shapes are polyhedra? Why are the other shapes not polyhedra? What can you say about their properties?*
* Provide groups of learners with a range of 3D shapes and ask them to identify as many properties of each 3D shape as they can.
* Play ‘20 questions’ in small groups or as a whole class. One learner secretly pulls a 3D shape name from a bag. The other learners ask questions that can only be answered 'yes' or 'no' to try to identify the shape. They are limited to asking a maximum of 20 questions before they must guess the shape. To encourage learners to use mathematical terminology, display key terms, e.g. ‘faces’, ‘vertices’ and ‘edges’.
 | 3D shapes (including some that are not polyhedra) such as cube, cuboid, cylinder, hemisphere, prisms, pyramids, sphere, tetrahedron3D shapes such as cube, cuboid, cylinder, hemisphere, prisms, pyramids, sphere, tetrahedronNames of 3D shapes on cards, e.g. cube, cuboid, cylinder, hemisphere, prism, pyramid, square-based pyramid, sphere, tetrahedronOpaque bag or box to draw cards from |
| 7Gs10 | Use a ruler, set square and protractor to:* measure and draw straight lines to the nearest millimetre
* measure and draw acute, obtuse and reflex angles to the nearest degree
* draw parallel and perpendicular lines
* construct a triangle given two sides and the included angle (SAS) or two angles and the included side (ASA)
* construct squares and rectangles
* construct regular polygons, given a side and internal angle.
 | * Provide learners with pre-drawn line segments for them to measure to the nearest millimetre. Learners then swap with a partner who checks their work. Discuss any discrepancies. *What can you do to make sure you are measuring as accurately as possible?*
* Learners work in pairs. One learner specifies a length in millimetres. Their partner draws a line segment of the given length, for the other learner to check. They then swap roles.
* Learners use pre-drawn angles to measure to the nearest degree. Learners swap with a partner who checks their work. Discuss any discrepancies. *What do you need to remember when using a protractor to make our measurements accurate?* *How can we measure reflex angles accurately?*
* Learners work in small groups. Give a number of degrees. Each learner draws an angle of the given size. Learners agree within their group who has drawn the most accurate angle. Repeat several times.
* Demonstrate using a ruler and a set square how to draw parallel and perpendicular lines.
* Learners work in small groups. Give a measurement in millimetres. Each learner draws parallel lines the given distance apart. Learners agree within their group who has drawn the most accurate parallel lines. *What advice do you have for drawing accurate parallel lines?* Repeat several times.

Carry out a similar activity for perpendicular lines. *What advice do you have for drawing accurate perpendicular lines?** Demonstrate how to construct a triangle given two sides and the angle.

 Learners take three cards – one ‘angle’ card and two ‘side’ cards. They construct the triangle matching the card measurements. A partner checks their length and angle measurements.Use the activities above, but swap to two angles and one side for ASA triangles. * Learners challenge each other by providing sides and angles of triangles for their partner to draw*.*
* Ask learners to use their skills and knowledge in drawing parallel and perpendicular lines and lines of given lengths to draw a square of side length 5 cm. *How did you make sure you drew an accurate square?* Repeat for a rectangle of given dimensions.
* Learners work in pairs. Ask them to use their skills and knowledge in drawing lines of a given length and angles to construct a regular pentagon with sides of length 6 cm and interior angle 108°. Learners take turns to draw the sides. Their partner checks the angle and length measurements before they draw the next side.

Discuss, e.g. *What skills and knowledge did you need to use? What did you find the most challenging part? Why do you think we used a set square to draw squares and rectangles, but a protractor for other regular polygons?*  | Pre-drawn line segmentsRulers (mm) Rulers (mm)Plain paperPre-drawn angles – a mixture of acute, obtuse and reflex anglesRulersProtractorsLarge ruler and set square Rulers (mm)Set squaresPlain paperLarge protractorLarge rulerTwo sets of cards: one set showing angle measurements and one set showing side measurementsProtractorsRulersProtractorsRulersPlain paperRulersSet squaresPlain paperRulersProtractors |

Problem Solving

Below are some possible activities linking the problem-solving strand to the knowledge and understanding learning objectives for the unit. To enable effective development of problem-solving skills, you should aim to include problem-solving opportunities in as many lessons as possible across the unit.

| **Activity ideas** | **Resources** |
| --- | --- |
| Learners draw and label these points: A(-1, 1) B(2,-1) C(-3, -1) D(5, -1) E(2, 2) F(1, -2) G(5, 2) H(2, 1)They then find the four points that are the vertices of: * a square
* a parallelogram
* a trapezium

They also give the vertices of different types of triangles, naming the triangles. | Coordinate grid or squared paperRulers |
| Learners design their own patterns using reflections, rotations and/or translations. They explain how they made their pattern. *Can you make the same pattern using different transformations?* | 2D shapes, paper and coloured pencilsor dynamic geometry software |
| To encourage learners’ investigative and visualisation skills, ask them to explore these activities in small groups:* Each group has 27 small cubes: three each of nine colours. They arrange the cubes to make a 3 by 3 by 3 cube so that each face of the bigger cube contains one of each colour.
* Learners imagine a three-dimensional version of noughts and crosses and consider the number of winning lines. *How many lines are there? Can you describe them?*
 | Small cubes |
| Learners work in pairs with a range of different polyhedra. They take one shape at a time and count its faces, edges and vertices. They record their findings in a table:

|  |  |  |
| --- | --- | --- |
| Faces | Edges | Vertices |
|  |  |  |
|  |  |  |
|  |  |  |

Learners discuss the tables, looking for any patterns in the numbers of faces, edges and vertices for polyhedra.  | Selection of polyhedra |
| Learners construct as many regular polygons as possible, given the side length and interior angle. | Plain paperRulersProtractorsAlternatively, learners could construct polygons using dynamic geometry software |