Scheme of Work – Stage 8 Mathematics

Introduction

This document is a scheme of work created by Cambridge as a suggested plan of delivery for Cambridge Lower Secondary Mathematics Stage 8. The learning objectives for the stage have been grouped into topic areas or ‘Units’.

This scheme of work assumes a term length of 10 weeks, with three terms per stage and three units per term. It has been based on the minimum length of a school year to allow flexibility. You should be able to add in more teaching time as necessary, to suit the pace of your learners and to fit the work comfortably into your own term times.

The units have been arranged in a recommended teaching order shown in the overview below. However, you are free to teach the units in any order that retains progression across the stage as your local requirements and resources dictate.

Some possible teaching and learning activities and resources are suggested for each knowledge and understanding learning objective. You should plan your lessons to include a range of activities that provide a progression of concepts and also reflect your context and the needs of your learners.

Teaching and learning in each unit should be underpinned by problem solving. For each unit, some possible activities are suggested which link the problem-solving strand to the knowledge and understanding learning objectives for the unit. To enable effective development of problem-solving skills, you should aim to include problem-solving opportunities in as many lessons as possible across the unit.

There is no obligation to follow the published Cambridge Scheme of Work in order to deliver Cambridge Lower Secondary. It has been created solely to provide an illustration of how teaching and learning might be planned across Stages 7–9. A step-by-step guide to creating your own scheme of work and implementing Cambridge Lower Secondary in your school can be found in the Cambridge Lower Secondary Teacher Guide available on the Cambridge Lower Secondary website. Blank templates are also available on the Cambridge Lower Secondary website for you to use if you wish.

Overview

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| --- | --- | --- |
| **Term 1** | **Term 2** | **Term 3** |
| Unit 1A Number, Calculation and Problem Solving | Unit 2A Number, Calculation and Problem Solving | Unit 3A Number, Calculation and Problem Solving |
| Unit 1B Algebra, Geometry and Problem Solving | Unit 2B Algebra, Geometry and Problem Solving | Unit 3B Algebra, Geometry, Measure and Problem Solving |
| Unit 1C Handling Data, Measure and Problem Solving | Unit 2C Handling Data, Measure and Problem Solving | Unit 3C Handling Data, Geometry, Measure and Problem Solving |

Unit 1A: Number, Calculation and Problem Solving

Number and Calculation

| Curriculum framework codes | Learning objectives | Activity ideas | Resources |
| --- | --- | --- | --- |
| 8Ni1 | Add, subtract, multiply and divide integers. | * Establish the meanings of ‘integer’ and ‘operation’. Write a range of numbers on the board, e.g. 3, 45, 6.78, 201, 3½. Ask learners to classify them under the headings: integers, fractions and decimals. * Starting with the number 5, each learner has to make a chain of 4 calculations using each of the operations in turn (×, +, -, ÷) to make another number which is an integer, e.g. 5 × 2 = 10, 10 + 14 = 24,   24 – 3 = 21, 21 ÷ 7 = 3.   * In pairs, learners create integer calculations with a given answer, e.g. ‘*The answer is 42. What is the calculation?*’   Encourage learners to use multi-step calculations. Ask e.g. *Can you find a calculation that uses addition and then division? If the division is ‘divide by 5’, what could the addition be?* (The addition would need to give a multiple of 5 and 42, such as 210.)   * Ask learners to find as many different integer calculations as possible using the digits of the year, e.g. for 2018:   201 × 8 = 1608  10 × 2 – 8 = 12  *What is the largest … smallest number you can make?* *Which operation will you try first to make the largest number? How will you try arranging the digits? Why?* |  |
| 8Np1 | Read and write positive integer powers of 10; multiply and divide integers and decimals by 0.1, 0.01. | * Introduce positive integer powers of 10 to learners by writing the following list on the board:   102 =  103 =  104 =  105 =  106 =  Learners discuss in pairs what these numbers might be. Establish that e.g. 104 = 10 × 10 × 10 × 10 and complete the list. *What patterns do you notice?* (Let learners think and discuss their ideas before you point them to 102 has two zeros, 103 has three zeros …). *What do you think 10100 would be?*   * Ask learners a series of true and false questions about multiplying and dividing by 0.1 and 0.01. Build in common misconceptions for learners to identify, e.g.   450 ÷ 0.1 = 45 (false, e.g. because dividing by a number less than 1 makes a number bigger)  0.01 × 0.2 = 0.02 (false, because multiplying by a number less than 1 makes a number smaller)   * Play a loop card game with the whole class using loop cards involving multiplying and dividing integers and decimals by 0.1 and 0.01. One learner reads out their calculation. The learner with the answer on their card reads out the answer and then their calculation, and so on. *What strategies did you use to carry out the calculation?*   Repeat to see if learners become faster.  Learners can work in pairs to create their own set of loop cards. | Pre-prepared set of loop cards (as shown in the examples below)   |  |  | | --- | --- | | The answer is 1.75 | What is  4 ÷ 0.1? |  |  |  | | --- | --- | | The answer is 20 | What is  8 × 0.01? | |
| 8Np2 | Order decimals, including measurements, making use of the =, ≠, > and < signs. | * Discuss =, ≠, > and < signs and ensure learners understand that ≠ represents ‘is not equal to’. Practise using the signs with familiar numbers, e.g.   Which two of the signs =, ≠, <, > could be used to make this number sentence true: 3 □ 5?   * Using three randomly chosen digit cards and a decimal point card, learners produce decimal statements that use =, ≠, > and < signs correctly. For example, if they have 4, 2 and 6 they could write:   62.4 > 42.6  4.62 < 46.2  6.24 = 6.24  2.64 ≠ 2.46  *How do you know which decimal is greater … less? Can you create a different statement using the same sign?*   * Learners find facts which involve measurements including decimals. Examples could include world records at athletics or masses of very small animals. They use these facts to produce statements that use =, ≠, > and < signs correctly. *What if your measurements are in different units, how do you know which is bigger … smaller?* | A set of 0–9 digit cards and a decimal point card for each learner.  Internet access for research  (Alternatively use real data from a class experiment, e.g. standing long jump distances) |
| 8Np3 | Round whole numbers to a positive integer power of 10, e.g. 10, 100, 1000, or decimals to the nearest whole number or one or two decimal places. | * Review rounding integers and rounding decimals to a given number of decimal places. Use integer and decimal data from real-life contexts, for example populations of cities, attendance at football matches, distances between towns, lengths/masses of animals. Ask, e.g. *How do you round to the nearest 10? Which digit do you need to look at?* * Give word problems based on calculations with real-life data including integers and decimals, e.g. *The attendance at a football match is 47 028. What is this to the nearest 10, 100, 1000?* | Pre-prepared word problems involving real life data (including integers and decimals) |
| 8Nc3  8Nf1 | Recall simple equivalent fractions, decimals and percentages.  Find equivalent fractions, decimals and percentages by converting between them. | * Learners work in pairs to list equivalent fractions, decimals and percentages they already know. Display this list for a future activity.   *How do you know that these are equivalent? If you know that these are equivalent fractions and decimals, can you use that fact to work out other equivalent fractions and decimals?*   * Show a familiar fraction and ask learners to deduce other equivalent decimals and percentages, e.g.   If 1/4 = 0.25 = 25%  then 1/8 = 0.125 = 12.5%  and 1/16 = 0.0625 = 6.25%   * Write 30% on the board. *What is the equivalent decimal?* (0.3) *What is the equivalent fraction?* (30/100 = 3/10).   Ask learners to write down other facts that they can deduce using these facts, e.g. 15% = 0.15 = 15/100 = 3/20.   * In pairs, learners play fraction, decimal and percentage snap. Each player has a set of cards with equivalent fractions, decimals or percentages e.g. 3/4 could be on one card and 75% on another). Learners take turns to put a card down on the table, and if two successive cards have the same value the first to say ‘snap’ wins those cards. | Fraction, decimal and percentage cards |
| 8Nf2 | Convert a fraction to a decimal using division; know that a recurring decimal is a fraction. | * Learners produce a list of equivalent fractions, decimals and percentages or use one from the earlier activity. Learners use a calculator to divide the numerator of each fraction by its denominator to see that this calculation always gives the equivalent decimal for a fraction. * Introduce the term ‘recurring decimal’ to describe decimals of the type 0.33333 …, 0.121212 …, 0.345345345 … etc. In small groups, learners use division on calculators to investigate fractions that are equivalent to recurring decimals. *Which fractions can you find that lead to recurring decimals? What patterns do you notice?* (e.g. prime number denominators other than 2 and 5 create recurring decimals). You could organise this activity so different groups explore fractions with different denominators, e.g. thirds, sixths, sevenths, ninths, elevenths. | Calculators  Calculators |
| 8Nf3 | Order fractions by writing with common denominators or dividing and converting to decimals. | * In pairs, learners discuss strategies for ordering given fractions, e.g. 7/16, 7/8, 3/4.   Discuss the strategies as a class, e.g.   * Writing the fractions with common denominators (drawing a diagram for support where needed) * Converting fractions to decimals * Using understanding of fractions with the same numerator, e.g. as sixteenths are smaller than eighths, 7/16 must be smaller than 7/8. * Learners work in pairs to produce sets of fractions for another pair to order. Check that learners can order the fractions themselves and justify their order before sharing with another pair. |  |
| 8Nc1 | Use known facts to derive new facts, e.g. given 20 × 38 = 760, work out 21 × 38. | * In pairs, learners write as many derived integer facts as they can from the given fact 20 × 38 = 760, e.g.   21 × 38 = 798  19 × 38 = 722  40 × 38 = 1520  200 × 38 = 7600  *How did you work out that new fact from the fact you were given?* *Now you have that new fact can you use it to derive another new fact?*   * Learners work in pairs. One learner writes down a fact they know and secretly writes down two derived facts. They tell their partner the starting fact. Their partner has to derive two new facts. They score one mark for each fact that is different from the secret facts. They then swap roles. * Learners write down a ‘fact chain’ of 10 facts from a given starting fact, e.g. 2 × 46 = 92, 20 × 46 = 920, 21 × 46 = 966, 21 × 23 = 483, 21 × 24 = 504. |  |
| 8Nc7 | Recall relationships between units of measurement. | * Provide learners with a set of pictures of objects (or real objects) and ask them how they would measure them. Ask them how many ways they could measure them (e.g. mass, height, capacity). Remind them of the work they did at Stage 7 on measure. * Ask learners to talk in pairs and list all the different units of measurement that they know and the relationships between them. Share the answers and make sure all commonly used units are included. * Explain that in some countries the unit ‘pounds’ is used instead of ‘kilograms’. Tell learners the approximate relationship between pounds and kilograms (1 kg = 2.2 lb), then ask learners to complete this table:  |  |  | | --- | --- | | **kilograms** | **pounds** | | 10 | 22 | | 5 |  | | 20 |  | |  | 40 | | 100 |  | |  | 2000 (1 ton) |  * In pairs, one learner draws a line and gives the measurement in one unit of measure; their partner has to give an equivalent length in another unit. They then repeat using different units of measure. | Set of pictures of real objects or real objects  Rulers |

Problem Solving

Below are some possible activities linking the problem-solving strand to the knowledge and understanding learning objectives for the unit. To enable effective development of problem-solving skills, you should aim to include problem-solving opportunities in as many lessons as possible across the unit.

| Activity ideas | Resources |
| --- | --- |
| Learners try to get as close as possible to a given answer with five given numbers and any of the operations +, –, × and ÷. *How can you use estimating to help you?* |  |
| Learners solve triangular arithmagons (see below) by finding the numbers that go in the circles. The numbers in the squares are the sum of the numbers in the circles on either side.    Learners deduce a rule to help them solve any triangular arithmagon. Learners make conjectures about when arithmagons cannot be solved. | Sets of arithmagons |
| Learners work in pairs reading statements written on cards, deciding if the statement is always true, sometimes true or never true, justifying their decision with an example, e.g.   * Numbers with more digits are greater in value. * If you add the same number to the top and bottom of a fraction, the fraction gets bigger in value. * If you divide the top and bottom of a fraction by the same number, the fraction gets smaller in value. | True and false number statements on cards |
| Learners attempt a wide range of word problems involving percentages, fractions and decimals such as:   * Would you rather have 75% of $200 or 5% of $2000? * Would you rather have 60% of 5 cakes or 20% of 20 cakes? * A football club has 20 members and 12 of these are boys. What percentage of the football club are boys? … girls? * In a sale, Shop A has reduced a pair of shoes originally priced at $80 by 25%. Shop B has reduced the same pair of shoes by 40%. Shop B’s original price was $100.   Which shop is cheaper? (Both are the same.) How did you work it out?   * A factory makes 100 000 wire coat hangers each day. Each coat hanger requires 87.4 cm of wire. How many kilometres of wire are used each day? What is the greatest number of hangers that can be made with 100 m of wire?   The factory makes smaller coat hangers from 85 cm of wire. How many more small hangers can it make from 100 m of wire?   * An airline specifies that hand luggage has to be such that length + width + height is less than 1 m. What dimensions of hand luggage would allow the largest space for its contents?   Ask learners to give reasons for their answers and to explain their strategies. | Calculators |

Unit 1B: Algebra, Geometry and Problem Solving

Algebra and Geometry

| Curriculum framework codes | Learning objectives | Activity ideas | Resources |
| --- | --- | --- | --- |
| 8Ae1 | Know that letters play different roles in equations, formulae and functions; know the meanings of ‘formula’ and ‘function’. | * Learners develop function machines and write their output as a formula.     *What function does this machine represent?* (Output = input × 4 – 3).   * Discuss examples of equations, formulae and functions. Ask learners to describe the different roles that the letters play in each. Establish the meanings of ‘function’ (where each input has one unique output) and ‘formula’. | Examples of equations, formulae and functions |
| 8As3 | Express simple functions algebraically and represent them in mappings. | * Remind learners about mappings.   Ask learners to describe the function for different mappings. Discuss how to express functions algebraically, e.g. f(*x*) = *x*2  **Input Output**  1  4  9  16  1  2  3  4   * Learners work in pairs with a set of function cards. One learner takes a card and draws the mapping diagram for the inputs 0, 1, 2, 3, 4. Their partner identifies the algebraic function from the mapping. | Pre-prepared mappings  Prepared cards showing simple algebraic functions |
| 8Ae2  8Ae4 | Know that algebraic operations, including brackets, follow the same order as arithmetic operations; use index notation for small positive integer powers.  Simplify or transform linear expressions with integer coefficients; collect like terms; multiply a single term over a bracket. | * Use variable cards to show how to collect like terms,  e.g. 3*a* + 2*b* + 2*a* – *b* (start with 3 cards labelled with *a;* add 2 cards labelled *b* …).   Ensure learners understand that only like terms can be added or subtracted.  *How can we simplify this expression by collecting like terms?*  4*x* + 7 + 3*x* – 3 – *x*   * Establish, for example, that:   In 8 – 3*x,* the multiplication is performed first.  In 4(*y* + 2), the brackets are evaluated first.  Ask learners to use these rules to evaluate expressions using a given value, such as:  3(*x* + 5) where *x* = 6  12 – (*n* – 3) where *n* = 8  *n*2(*n* – *p*) where *n* = 9 and *p* = 3  *n*3(*n4* – *p*) where *n* = 2 and *p* = 7  Remind learners about index notation. In the example above they can use *n*3 × *n*4= *n*3+4 = *n*7to help with the calculation.   * Demonstrate that 7 × 36 = 7(30 + 6) = 7 × 30 + 7 × 6 (distributive law) can be represented as *a*(*b* + *c*) = *ab* + *ac*.   Also demonstrate that 7 × 49 = 7(50 – 1) = 7 × 50 – 7 × 1 can be represented as *a*(*b* – *c*) = *ab* – *ac*.  Learners repeat this process with similar expressions.  Ask learners to use these rules to expand expressions such as:  3(*x* + 5);  12 – (*n* – 3)  *n*2(*n* – *p*)  Use the initial numerical subtraction example to reinforce that a negative number multiplied by a positive number is a negative number.   * Learners expand algebraic expressions such as a(*b* + *c*) and   *z*(*x* – *y*).   * Ask learners to expand and simplify expressions such as 4(*a* + 2*b*) – 2*a*(2*a2* + *b*). * Ask learners to write different equivalent expressions for the total length of the lines in this diagram.   Learners should simplify each expression as far as possible.  *What did you discover?* (They all simplify to 4(*A + B*).)  A  B   |  |  |  | | --- | --- | --- | |  |  |  | |  |  |  | |  |  |  | | Cards showing variables |
| 8Ae3 | Construct linear expressions. | * Ask learners to show that this is a magic square (all rows and columns have the same total).  |  |  |  | | --- | --- | --- | | *a* – *b* | *a* + *b* – *c* | *a* – *c* | | *a* + *b* – *c* | *a* | *a* – *b* – *c* | | *a* – *c* | *a* – *b* – *c* | *a* + *b* |   *Explain your strategy for solving this problem.* Discuss the expressions for the different rows and columns and how they can be simplified.  By substituting numbers for *a*, *b* and *c* learners, form a numerical magic square.   * Ask learners to solve this pyramid (the expression in each cell is calculated by adding the two expressions below):  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | |  |  | ? | |  |  | |  | 3*a* + *b*2 | | 2*a* + 3*b*2 | |  | | ? | | *b*2 | | ? | |   *Explain your strategies for finding the missing expressions.*  By substituting numbers for *a* and *b*, learners form a numerical pyramid to check their answers.   * Provide learners with definitions of four variables: * *a* is the number of apples I bought * *b* is the number of bananas I bought * *x* is the cost of one apple * *y* is the cost of one banana.   Give learners a set of cards describing a situation (e.g. the number of apples and bananas I bought altogether; the cost of all the apples I bought). Give them a set of cards with equivalent expressions (e.g. *a + b; ax).* Ask them to match the cards. | Sets of matching cards:  Set 1: situations  Set 2: expressions representing situations |
| 8Gs1 | Know that if two 2D shapes are congruent, corresponding sides and angles are equal. | * Ask learners to define the term ‘congruent’. Use paper shapes to show that congruent shapes have corresponding sides and angles equal, i.e. they can fit exactly on top of each other. * Learners are given a selection of cardboard shapes; they find those that are congruent. * Learners work in pairs drawing congruent shapes. One learner draws a shape; the partner has to draw another shape which is congruent. Encourage learners to draw the shapes in different orientations. | Irregular congruent paper shapes  Selection of cardboard shapes, some of which are congruent  Squared paper |
| 8Gs2 | Classify quadrilaterals according to their properties, including diagonal properties. | * Learners sort quadrilaterals into sets (e.g. using Venn diagrams) with similar properties, e.g. those that have right angles, parallel sides, diagonal properties. Ensure learners understand the meaning of the term ‘diagonal’ for 2D shapes (a straight line from one vertex to a non-adjacent vertex). * Learners explore the diagonal properties of quadrilaterals using pre-drawn quadrilaterals. *How many diagonals does a quadrilateral have? Which quadrilaterals have diagonals that bisect each other? …diagonals that are equal in length? … diagonals that are perpendicular?* * Ask learners to work in groups to create posters which give all the properties of these quadrilaterals: square, rectangle, rhombus, parallelogram, trapezium, kite. | Card/plastic quadrilaterals)  Large circles for Venn diagrams  Pre-drawn quadrilaterals  Large sheets of paper  Marker pens |
| 8Gs3 | Know that the longest side of a right-angled triangle is called the hypotenuse. | * Explain that the longest side of a right-angled triangle is called ‘the hypotenuse’. In pairs, learners sketch a range of right-angled triangles in different orientations (i.e. not always with a horizontal base). Their partners identify the hypotenuse in each triangle. *How are these two triangles the same … different?* * Learners use different circular pinboards to explore making triangles on, e.g.     *Is it possible draw a right-angled triangle?* *… an isosceles triangle? … an equilateral triangle? Identify the hypotenuse where possible.* | Pinboards and elastic bands, or paper circles with different numbers of points drawn on them |
| 8Gs4 | Identify alternate angles and corresponding angles. | * Display this diagram with one pair of corresponding and alternate angles labelled:     alternate angles  corresponding angles  Check learners’ understanding of the relative positions of different types of angles. *Which other angles are corresponding angles? … alternate angles? Why?*   * *What are parallel lines? What is a transversal?* In pairs, learners create a poster to illustrate and define alternate angles and corresponding angles on a pair of parallel lines. * Learners draw different transversals and parallels and use a protractor to show that pairs of alternate and corresponding angles are equal. *What happens when the two lines being crossed by the transversal are not parallel?* | Large sheets of paper  Marker pens  Rulers  Protractors |
| 8Gp1 | Find the midpoint of the line segment AB, given the coordinates of points A and B. | * Model the construction of the midpoint and perpendicular bisector of a line segment. *Why does this construction work?* (You are finding points that are equidistant from each end of the line segment and all points on a perpendicular bisector are equidistant from each end of a line segment.)   Learners practise the process using a range of line segments in different orientations. Give learners the coordinates of points A and B. Learners draw the line segment AB on a coordinate grid. Learners find the midpoint of AB using the construction.   * Draw a line segment AB on a coordinate grid and label the points A and B with their coordinates. *How can you find the midpoint without using the construction?* (The *x*-coordinate of the midpoint (*mp*) is half the sum of the *x-*coordinates. The *y*-coordinate of the midpoint is half the sum of the *y*-coordinates.)   Give learners the coordinates of the end points of different line segments. Ask learners to calculate the midpoint of each line segment. | Large straight edge  Large pair of compasses  Coordinate grids or squared paper  Pairs of compasses  Straight edges (e.g. rulers)  Large coordinate grid |

Problem Solving

Below are some possible activities linking the problem-solving strand to the knowledge and understanding learning objectives for the unit. To enable effective development of problem-solving skills, you should aim to include problem-solving opportunities in as many lessons as possible across the unit.

| Activity ideas | Resources |
| --- | --- |
| Learners develop pyramid puzzles like the one below for a partner to solve. They check their answers by substituting numbers for the letters. Learners could develop pyramids with more than three layers.   |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | |  |  | ? | |  |  | |  | 3*a* + *b*2 | | 2*a* + 3*b*2 | |  | | ? | | *b*2 | | ? | | |  |
| “An isosceles triangle is made up of two right-angled triangles.”  “A rhombus is a parallelogram, but a parallelogram is not a rhombus.”  Learners decide whether these statements are true or false. *How do you know?* |  |
| Ask learners: *What is the maximum number of right angles a hexagon can have? Investigate for other plane shapes.* |  |
| Learners use angle facts to prove that opposite angles of a parallelogram are equal. |  |
| Learners use a 3 × 3 pinboard to find all the possible quadrilaterals (16) that can be made and then classify them according to their properties. Ask them to explain their reasoning. | Pinboards and elastics bands, or dotty paper |

Unit 1C: Handling Data, Measure and Problem Solving

Handling Data and Measure

| Curriculum framework codes | Learning objectives | Activity ideas | Resources |
| --- | --- | --- | --- |
| 8Ml1 | Choose suitable units of measurement to estimate, measure, calculate and solve problems in a range of contexts, including units of mass, length, area, volume or capacity. | * Learners are asked to find a missing dimension given the volume and the other dimensions, e.g. *A cuboid is 2cm wide and 7cm high, and its volume is 42cm3 – how long is it?* * In pairs, one learner thinks of something that can be measured, and the other learner has to give the relevant units of measurement. * Give problems such as:   How far would you walk if you walked for a million paces?  How high would 1000 chairs be if they were stacked on top of each other?  If the average breath takes in 0.5 litres of air, how many litres do you breathe in a year?   * Learners research animals with the smallest and the largest mass. They calculate how many times ‘heavier’ one animal is than another. |  |
| 8Dc1  8Dc2 | Identify and collect data to answer a question; select the method of collection, sample size and degree of accuracy needed for measurements.  Know the difference between discrete and continuous data. | * Discuss the difference between discrete data and continuous data. Learners give each other a set of data; they have to decide whether it would be discrete or continuous. * Learners count the colours of cars in a car park, producing a frequency table. *Is this data discrete or continuous?* * Learners plan and carry out a meaningful survey in small groups, e.g. to find out how often and how people travel to a shopping mall. * Learners decide how to answer a question such as ‘Does the type of a cup affect the time taken for tea to cool?’ or investigate a statement such as ‘Tall people have large feet.’ Learners decide what data they need to collect and how to collect it, considering sample size and degree of accuracy for measures. |  |
|  |  |  |
| 8Dc3 | Construct and use:   * frequency tables with given equal class intervals to gather continuous data * two-way tables to record discrete data | * Learners work in small groups taking on the role of a research team for a manager of a shoe shop who needs to gather information to help him decide what stock he should buy. * They construct a frequency table to record the **shoe size** of everyone in their group or the class. * They measure the **length** of everyone’s feet and construct a frequency table, deciding on class intervals.   *Is the data discrete or continuous?* *Why?* Help learners to understand the difference between discrete and continuous data. Establish that the length data they have collected is continuous data because, e.g. feet can measure 250 mm, 255 mm or any length in between. The size data is discrete data because e.g. sizes 40 and 41 mean something but size 40.3 does not. Discuss if having sizes such as 40.5 changes anything. *Is it still discrete data?*  *How can you make sure your class intervals for the foot lengths cover all possible measurements?* Support learners in using inequalities for continuous class intervals, e.g. 250 mm ≤ *l* < 260 mm.   * Learners also complete a two-way table showing the preferences for styles of shoes for boys and girls in the class. Learners should decide on the ‘styles’ of footwear they will use:  |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | --- | |  | Trainers | Boots | High heels | Low heels | | Total | | Boys |  |  |  |  |  | | | Girls |  |  |  |  |  | | | Total |  |  |  |  |  | |   Explain that this type of frequency table (with two types of categories) is called a two-way table. *Why would the data you have collected today be useful for shoe shop managers?* (e.g. Shoe sizes will help them to order the right proportion of e.g. size 38 shoes compared to other sizes.)   * Learners design and use a frequency table to record the method and distances learners travel to school, using distance class intervals such 0.5 km, so 0 km ≤ *l* < 0.5 km, 0.5 km ≤ *l* < 1 km etc. |  |
| 8Dp1 | Calculate statistics for sets of discrete and continuous data; recognise when to use the range, mean, median and mode and, for grouped data, the modal class. | * In groups of four, each learner calculates the range, mean, median and mode / modal class of a set of data. Make sure to include examples of discrete and continuous data, and grouped data. Learners then discuss what each statistic shows in the context of the data. (e.g. the modal shoe size shows which shoe size is most common for your age group) * Learners repeat with a different sample e.g. the number of words/letters in a 100-word passage from different newspapers.   *Which statistics are most useful for each set of data? Why?* | Sets of sample data |
| 8Di1 | Interpret tables, graphs and diagrams for discrete and continuous data, and draw conclusions, relating statistics and findings to the original question. | * Learners analyse the results of a survey they have conducted and check the validity of the hypothesis. Other learners draw frequency diagrams to illustrate the data. *Think about the class intervals for the foot length data. How can you adapt a bar chart to show that every foot length is covered?* Establish that you can close up the bars. You can also label the boundaries of the class intervals rather than the class intervals themselves. * Groups discuss (in the role of shoe shop managers from the activity in 8Dc3) how they will decide on stock for their shop. They justify their decisions using their statistics, charts and graphs. If time allows, each group presents their conclusions to the whole class. *What extra research would a real shop manager need to do?* (e.g. survey the shoe sizes of a bigger and wider sample) |  |
| 8Db1 | Know that if the probability of an event occurring is *p*, then the probability of it not occurring is  1 – *p*. | * Display three large sheets of paper labelled ‘Impossible’ ‘Uncertain’ ‘Certain’. In pairs, learners think of at least two events that they could put into each category. Take feedback and list all the events on the sheets of paper. * Record learners’ suggestions of likelihood statements for ‘uncertain’ events (e.g. a small chance, almost certain, quite certain, not a great chance, more than likely, fairly likely, a good chance, extremely likely, very likely, a very good chance). Learners attach ‘uncertain’ events (e.g. from the list created in the activity above) to appropriate statements. * Remind learners that a probability scale runs from 0 (impossible) to 1 (certain). Draw a 0–1 scale. Give an ‘uncertain’ event (e.g. from the activity above). In pairs, learners discuss a likelihood statement and a probability value for it. *How did you decide on your probability value?*   Pairs then take it in turns to draw a cross on the probability line where they think the probability will fall. This process models ‘experimental’ probability with a tendency towards a value.   * Use questioning to introduce that if the probability of an event occurring is *p*, then the probability of it not occurring is 1 – *p*:   *What is the probability of getting the number/tail side when you toss a coin? What is the probability of tossing the non-number/head side?*  *What do you notice about the sum of the probabilities? Why is this?* (It is certain that you will toss either a number/head or a non-number/tail, so the probabilities add to 1.)  *If there is a 60% chance of it raining tomorrow, what is the probability that it won’t rain?* | Large sheets of paper |
| 8Db2 | Find probabilities based on equally likely outcomes in practical contexts. | * In pairs, learners carry out an experiment for which the outcomes are unknown. For example, they use bags of sweets/beads with different colours and find out what the probability of picking a red sweet is by picking out a sweet and replacing it. (Each bag contains 25 sweets). Each pair carries this out 20 times, recording the results, and calculates their experimental probability of picking a red sweet.   *Why are we replacing the chosen sweet each time?* (If the sweet is taken away, the total number changes so the probability of picking a red sweet next time changes too.) *What is the experimental probability of picking a red sweet? Why?* Compare the probabilities found by each pair. *Why are all the probabilities different?* (e.g. There are likely to be different numbers of red sweets in different packets.)  Learners count their total number of sweets and their red sweets. *What is the theoretical probability of picking a red sweet? Why? What is the probability of* ***not*** *picking a red sweet? Does the experimental probability match the theoretical probability? Why?*     * Ask learners: *What is the hardest number to get if you throw a die?* In pairs, learners carry out an experiment to test their theories. They throw a die 36 times and record the results. Each pair uses their results to calculate the experimental probability of throwing a 6.   Combine the results from the whole class to calculate the experimental probability for the whole class. Discuss why the result is not exactly 1/6. | Bags of sweets or beads Dice |

Problem Solving

Below are some possible activities linking the problem-solving strand to the knowledge and understanding learning objectives for the unit. To enable effective development of problem-solving skills, you should aim to include problem-solving opportunities in as many lessons as possible across the unit.

| Activity ideas | Resources |
| --- | --- |
| Learners find the missing data items from a data set given one or more of the statistics, e.g. *If the mean of 6 numbers is 4 and the mode is 3 what could the numbers be?* |  |
| Nisha and John play three games. Their scores have the same mean. The range of Nisha’s scores is twice that of John’s scores.   |  |  |  |  | | --- | --- | --- | --- | |  | Game 1 | Game 2 | Game 3 | | Nisha’s score |  | 40 |  | | John’s score | 35 | 40 | 45 |   Learners complete the table above. |  |
| The names of all Stage 8 learners, teachers and lunch time staff are put in to a box. One name is pulled out at random. A learner says:  “There are 3 choices: it could be a learner, a teacher or lunch time staff. The probability of it being a learner is 1/3.”  *Is the learner correct? Why?* |  |
| Learners decide whether spinner games are fair, e.g.  Bill and Tula play a game. They spin a pointer a lot of times. If it stops on an odd number, Bill scores 2 points. If it stops on an even number Tula gets 3 points. *Is it a fair game? Why?*  1  3  5  4  2 |  |

Unit 2A: Number, Calculation and Problem Solving

Number and Calculation

| Curriculum framework codes | Learning objectives | Activity ideas | Resources |
| --- | --- | --- | --- |
| 8Ni2 | Identify and use multiples, factors, common factors, highest common factors, lowest common multiples and primes; write a number in terms of its prime factors, e.g.  500 = 22 × 53. | * In pairs, learners pick two digit cards at random and make the largest number they can. Their partner then lists all the factors. *How are you trying to make sure you find all the possible factors?* (e.g. decide whether 1 is a factor, then 2, then 3; find factor pairs)   *How can you decide whether … is a factor?* (e.g. using tests of divisibility) If both partners agree they then swap roles.   * Ask learners to list all the factors of 36 and 48. *What are the common factors?* *What is the highest common factor (the largest factor that both numbers have in common)?* (12)  Repeat for other pairs of numbers. * Learners solve word problems involving lowest common multiples (LCMs) and highest common factors (HCFs). e.g. * Jill and Sarah are putting books on shelves in the library. Jill puts 5 books at a time on the shelf, Sarah puts 6. They both stack the same number of books. What is the smallest they could have stacked? * Pencils come in packages of 10. Rubbers come in packages of 12. Philip wants to purchase the smallest number of pencils and rubbers so that he will have exactly 1 rubber per pencil. How many packages of pencils and rubbers should Philip buy? * Learners make up similar problems for each other. * Model how to use a factor tree and write a number in terms of its prime factors. An example is:      * Learners generate two two-digit numbers using their digit cards. They create a factor tree for each number and write each number in terms of its prime factors. Then they list the common factors, the highest common factor and the lowest common multiple of the two numbers. | A set of 0–9 digit cards for each pair of learners  A set of 0–9 digit cards for each pair of learners |
| 8Nf4 | Add and subtract fractions and mixed numbers; calculate fractions of quantities (fraction answers); multiply and divide an integer by a fraction. | * Learners use their understanding of equivalent fractions to carry out calculations such as 1/4 + 11/8 and 11/12 – 3/6 and with mixed numbers such as  11/4 + 2 11/8.   They work in pairs with one learner giving their partner instructions to carry out the calculation.   * Ask learners to describe the strategy that they use to add and subtract fractions. They should realise that you have to convert to equivalent fractions with the same denominator to carry out an addition or subtraction calculation. Link this with the work on LCMs and HCFs. * Learners are given answers to addition and subtraction problems involving fractions and asked to:   *Find two fractions (which are not eighths) that add up to 7/8.*  *Find two fractions (which are not quarters) with a difference of ¼.*  *Find two fractions (which are not both sixths) that add up to 25/6.*  *Find two fractions (which are not both quarters) that have a difference of 1¼.*  *How did you work it out? Can you find two different fractions?*   * *Ask learners what they think 3 ÷ 1/4 means?*   (How many quarters are there in 3 wholes? If learners answer 3 divided by one fourth, accept the answer but draw their attention to what the division means.)  Use a diagram to demonstrate solving this. There are 4 quarters in 1 so there are 3 × 4 quarters in 3:   |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | | 1 | | 1 | | 1 | | | 1/4 | 1/4 |  |  |  |  | | 1/4 | 1/4 |  |  |  |  |   So 3 ÷ ¼ = 3 × 4 = 12.   * In pairs learners use digit cards to generate and solve calculations of the form:      ÷ 1/🞏  *Can you describe a rule for dividing an integer by a fraction with numerator 1?* (Multiply the integer by the denominator of the fraction.)   * Ask learners to solve this problem:     My friend has 60 sweets. Each day he keeps a fraction of the sweets, gives the rest away, and then eats one. These are the fractions he keeps:  Day 1 – 3/4  Day 2 – 7/11  Day 3 – 5/9  Day 4 – 2/7  Day 5 – 2/3.  How many did he have left at the end? How did you work out how many he kept each time?  *How would you calculate ¾ of 60*? (e.g. find one quarter by dividing by 4 and then multiply by 3) | Fraction cards used to generate random questions  A set of 0–9 digit cards for each pair of learners |
| 8Nf5 | Calculate and solve problems involving percentages of quantities and percentage increases or decreases; express one given number as a fraction or percentage of another. | * Ask ‘Would you rather …?’ questions, e.g. * *Would you rather have 80% of $400.00 or 20% of $4000.00?* * *Would you rather have 60% of 5 pieces of a cake or 30% of 10 pieces of the same cake?* Ask learners to give reasons for their answers and to explain their strategies. Learners make up similar questions for each other. Questions should have answers which are close to each other to make the comparisons less obvious. * Learners solve the following problem individually and then share their answers and strategies with a partner:   A football club has 20 members and 12 of these are boys. What percentage of the football club are boys? … girls?  *Did you both use the same strategy? If not, what did you do differently?* Establish that one strategy is to calculate the fractions first and then convert to percentages.   * Learners use data from the class or from the whole school to carry out calculations involving fractions and percentages. For example:   *What fraction of the school population is male? What percentage of the class is female?*   * Learners solve a percentage increase in pairs:   I have $500. I increase my total by 15% each year. How many years will it take me to double my money?  (Learners should round to the nearest dollar after each calculation.)  *How can you calculate 15% of an amount mentally? … with a calculator? What steps did you use to find the number of years?*   * In pairs, learners solve a percentage decrease problem.   In a sale Shop A is reducing a pair of shoes originally priced at $160 by 25%.  Shop B is reducing the same pair of shoes by 40%. Shop B’s original price was $200.  *Which shop is cheaper?* (Both are the same.) *How did you work it out?*  Ask learners to make up other pairs of reductions which make the sale prices equal.   * Learners play percentage increase and decrease loop cards. *How did you carry out that calculation?* * Learners create their own set of loop cards to play at home. *How do you know that your loop cards will make a complete loop?* | Sale literature from any shop, flyers etc.  Pre-prepared set of loop cards involving percentage increases and decreases  (Optional) Template for learners to create their own loop cards  Example of a percentage increase and decrease loop card set   |  |  | | --- | --- | | Decrease 20 by 10% | 36 | | What is 20% of 125? | 18 | | Increase 8 by 12.5% | 25 | | Increase 20 by 50% | 9 | | Decrease 80 by 15% | 30 | | What is 50% of 72? | 68 | |
| 8Nf6 | Use equivalent fractions, decimals and percentages to compare different quantities. | * Learners use labels on food packaging to carry out calculations using nutritional data and fractions and percentages, e.g. * What percentage of the food is sugar? * What fraction is this? * How many grams of sugar are there in the whole packet? Half of packet? * Learners answer questions such as:   A factory rejects 6 out of every 300 items, what percentage is this? How do you represent it as a decimal?   * Learners decide in pairs which are the better buys: * A 400g packet of biscuits costing 52 cents or a pack with 400g plus 25% at 57 cents. * A shop sells blank CDs in packs of 8 for $3.50 or in packs of 30 for $11.00.   Learners make up similar questions for a partner.   * Learners challenge each other by one giving a fraction of an amount. Their partner has to give a different fraction of a different amount that will be the same, e.g. 0.5 of 200 and 10% of 1000. They then represent the fraction with decimals and percentages. | Food items/packaging showing nutritional information |
| 8Nc2 | Recall squares to 20 × 20, cubes to 5 × 5 × 5, and corresponding roots. | * Learners re-order the numbers 1 to 17 so that each adjacent pair adds up to a square number.They have to explain orally what strategies they used to solve this problem? *Can you use all the numbers?* * Some numbers are equal to the sum of two squares, e.g. 52 = 16 + 36. Learners work in pairs and take turns telling each other numbers less than 100 that are the sum of two squares. * Learners test each other orally on square numbers and square roots, and cubes and cube roots. They can create a set of loop cards to help. |  |
| 8Nc5 | Use known facts and place value to multiply and divide simple decimals, e.g. 0.07 × 9, 2.4 ÷ 3. | * Learners describe patterns in multiplication by powers of 10, e.g. 12 × 10, 12 × 1, 12 × 0.1, etc. * In pairs, learners tell each other as many derived facts as possible from the given fact 7 × 9 = 63 (or any other from teacher’s or learners’ choice). Each fact should use decimals and they should include division as well as multiplication, e.g.   7 × 9 = 63 63 ÷ 9 = 7  0.7 × 9 = 6.3 6.3 ÷ 9 = 0.7  0.07 × 9 = 0.63 0.63 ÷ 0.07 = 9  1.4 × 9 = 12.6 12.6 ÷ 9 = 1.4  *How do you know that that new fact is correct? What adjustments have you carried out?* Learners may need to use calculators to check some answers.   * Learners work in pairs. One learner writes down a fact they know involving multiplication and division by decimals, and secretly writes down two derived facts. They tell their partner the starting fact. Their partner has to say two new facts. They score one mark for each fact that is different from the secret facts. They then swap roles.   *How do you know that the new fact is correct? What adjustments have you carried out?* Learners may need to use calculators to check some answers. | Calculators  Calculators |
| 8Nc6 | Use known facts and place value to calculate simple fractions and percentages of quantities. | * Learners work in pairs to create spider diagrams using equivalent amounts expressed as fractions, decimals and percentages of quantities. One of the options should be blank and one learner tell their peer what the answer would be. For example, start with 12 in the centre, then one learner adds ‘branches’ for 25% of 48 and ¼ of 48. The other learner needs to say that the one that is missing is 0.25 × 48 … Then they swap roles. * Learners find complex percentages such as 17½% of quantities by finding 10%, halving for 5%, halving for 2½% then adding together. In pairs or groups, they explain each other which strategy they used. * Ask learners to say different ways in which a percentage of a quantity can be found,   e.g. 35% = 10% + 10% + 10% + 5%. | Large sheets of paper for spider diagrams |
| 8Nc8 | Solve simple word problems including direct proportion problems. | * Learners find the cost of 6 pizzas if 8 cost $16. * Learners solve problems such as: * Coffee is made from two types of beans, Java and Vietnam, in the ratio 2:3. How much of each type of bean will be needed to make 500 kg of coffee? * Lotto winnings are divided between two friends in the ratio of 2:5. If Anil got the smaller amount of $1000, what was the total lotto winnings? * A cake recipe requires 1 cup of sugar for 2 of flour. How much sugar and flower you need to bake 3 cakes? Half of the recipe? * Ask learners to convert $100 to various other currencies using the unitary method (See 8Nf8 for explanation). Ask learners*: How can you find the answer by the unitary method?* * Choose learners to present their strategies and solutions to the rest of the class. Include the unitary method in your discussions. |  |
| 8Nc11 | Consolidate adding and subtracting integers and decimals, including numbers with differing numbers of decimal places. | * Learners work in pairs to create a set of revision notes that explain how to carry out written calculations to add and subtract integers and decimals with differing numbers of places. *How will you decide which examples to include?* Select a pair at random to share their notes with the rest of the class. The class gives feedback on this presentation. * Learners are given some completed calculations (with integers and decimals), each with an error or errors; they have to find the error then describe how they would rectify it not only changing the result. * In pairs, learners create a set of 10 questions involving adding and subtracting decimals (with equal and differing numbers of decimal places). They exchange their questions and then check and mark each other’s answers. *What kinds of errors were most common? How will you make sure you don’t make these errors in the future?* | Set of completed calculations each with an error, e.g.  5.2 + 3.04 = 8.6  405 – 35 = 100 |
| 8Nc12 | Divide integers and decimals by a single-digit number, continuing the division to a specified number of decimal places, e.g. 68 ÷ 7. | * Review the written method of division for division by single-digit numbers. Discuss both integer and decimal dividends which divide exactly, e.g.   1. 3 8  3 ) 4. 11 24  *What about 4.13 ÷ 3?*   * Model the division, extending an answer to 4 or 5 decimal places.   1 . 3 7 6 6 …  3 ) 4 .11 23 20 20 2…  Also find the answer using a calculator.  Explain that rounded answers are usually accurate enough. *What is this answer, rounded to 2 decimal places?*   * Learners create five different division calculations using four-digit cards. They should all involve division by a single-digit number but should include a range of dividends with 1 and 2 decimal places. Each division should be answered to 2 decimal places. * Learners explore dividing two-digit integers by 3, 6 and 9. All answers should be given to 3 decimal places.   *How can you check your answers? Do you need to do a written division to divide by 6 … 9?* | Calculators  Set of 1 to 9 digit cards |

Problem Solving

Below are some possible activities linking the problem-solving strand to the knowledge and understanding learning objectives for the unit. To enable effective development of problem-solving skills, you should aim to include problem-solving opportunities in as many lessons as possible across the unit.

| Activity ideas | Resources |
| --- | --- |
| Prove that the sum of any five consecutive numbers is divisible by 5. Make conjectures about the sum of different numbers of consecutive numbers. Does the hypothesis hold for consecutive odd numbers? |  |
| Choose a 2 by 2 square from a multiplication grid, e.g.   |  |  | | --- | --- | | 8 | 12 | | 10 | 15 |   Multiply the opposite numbers.  8 × 15 = 120  10 × 12 = 120  *What do you notice? Is this true for all 2 by 2 squares on the grid? Why?*  *What about larger squares? Rectangles?* | Multiplication grids |
| Learners find two numbers with:  a sum of 0.8 and a product of 0.15  a sum of -11 and a product of 28  a difference of 4 and a quotient of 3 |  |
| To make orange paint, you mix 13 litres of yellow paint with 6 litres of red paint and 1 litre of white paint. *How many litres of each colour do you need to make 10 litres of orange paint?* |  |
| Give learners missing number written additions and subtractions to solve, e.g.  9 4 3 🞏  + 3 7 🞏 8  1 🞏 🞏 1 1    5 7 8 1 🞏 1  – 🞏 🞏 4 1 2 🞏  3 8 🞏 🞏 7 2 |  |
| Learners state whether these statements are true or false, and justify their answers.  4.37 × 0.03 = 1.311  5.2 × 0.6 = 3.12  53.1 ÷ 0.5 = 26.55 |  |

Unit 2B: Algebra, Geometry and Problem Solving

Algebra and Geometry

| **Curriculum framework codes** | **Learning objectives** | **Activity ideas** | **Resources** | |
| --- | --- | --- | --- | --- |
| 8Ae5 | Derive and use simple formulae, e.g. to convert degrees Celsius (°C) to degrees Fahrenheit (°F). | * Ask questions involving average speed:   *If I travel for 2 hours at 50 km per hour how many kilometres do I cover?*  *If I drive 100 km at 40 km per hour how long does it take me?*  Use the discussion of the calculations to derive the formula:  where s = average speed  d = distance covered  t = time taken  Discuss how to find the distance or the time given the other two values. Learners work in pairs and set each other simple word problems using the formula.   * Learners plot given pairs of temperatures (0, 32) (100, 212) and draw a line through data points – they discuss how to develop a formula to convert from Celsius to Fahrenheit. * Ask if learners know any other examples of formulae. If not, use cooking formulae such as:   Cooking time for a chicken =  20 minutes per kg + 20 minutes. | Squared paper | |
| 8Ae6 | Substitute positive and negative integers into formulae, linear expressions and expressions involving small powers, e.g. 3*x*2 + 4 or 2*x*3, including examples that lead to an equation to solve. | * Learners decide whether each of the following statements is sometimes true, always true or never true.   3 + *a* = *a* + 3  2 – *a* = *a* – 2  *ab* = *ba*  *a* ÷ *b* = *b* ÷ *a*  (*a* + 3)2 = *a*2 + 32  2*a*3 = 23 + *a*3  Learners explain and justify their conclusions (e.g. by substituting numbers into each statement).   * Learners create their own equations involving at least one square and/or a cube. A partner checks each equation by substituting positive and negative integers. * Ask learners questions like: * If *a* + *b* = 25 what might *a* and *b* represent? * If *c* – 2*d* = 32 what might *c* and *d* represent?   Do the same for known formulae:   * If the area of a triangle is 20cm2, what might be h and b? |  | |
| 8As4 | Construct tables of values and use all four quadrants to plot the graphs of linear functions, where *y* is given explicitly in terms of *x*; recognise that equations of the form *y* = *mx* + *c* correspond to straight-line graphs. | * Ask learners to give you instructions to draw a 4-quadrant set of axes from -10 to +10. Make sure all the key features are identified (e.g. *x*-axis, *y*-axis, origin). *What key features should appear on all sets of axes?*   Using the function *y* = 2*x*, ask learners for the value of *y* if *x* = 1, *x* = 2, *x* = -2, etc.  They then construct a table of values for *x* = -5 to *x* = 5, and plot the coordinate pairs on the Cartesian plane. They join the points to make a graph. Learners repeat for *y* = 2*x* + 1 and *y* = 2*x* + 2.  *What is the same about all the lines? What is different? How can you describe the coordinates of any point on the line? What is the equation of the line?*   * Learners work in pairs to explore graphs (graphs on a set of axes, or equations or coordinate pairs) for a range of linear functions. Some should have the same gradient and some the same intercept. They represent each graph as a table and deduce the equations for the lines. *What patterns do you notice in the tables? … in their graphs?*   Discuss learners' findings and establish that all straight-line graphs can be written in the form *y* = m*x* + c. |  | |
| 8Gs7 | Draw simple nets of solids, e.g. cuboid, regular tetrahedron, square-based pyramid, triangular prism. | * Use ruler and compasses to construct the net of each solid, colour and make them into a mobile. * Give learners boxes which are common 3D shapes, including cuboid, regular tetrahedron, square-based pyramid, and triangular prism. Learners draw the nets of the boxes. They test the accuracy of their nets by folding. At the end, they open the boxes to compare the actual net with their nets. * Learners find different ways to draw nets of a particular solid. * Learners identify from a set of pre-drawn nets which will and which will not make a solid. | Cardboard boxes (which can be dismantled): cuboid, regular tetrahedron, square-based pyramid, triangular prism  Paper  Rulers  Pairs of compasses  Set of pre-drawn nets, some of which will not fold to make a solid. | |
| 8Ma3 | Use simple nets of solids to work out their surface areas. | * Learners work in small groups. Give each group a different object. They design a package they could use if they were selling the object. They design their package and calculate its surface area using its net. (For more complicated nets they may need to approximate the surface area. If so, ask learners to explain how they have made their approximation.)   6  4  3   * Learners find the surface area of a 3D shape made when a cuboid is cut in half by a single vertical cut, using their knowledge of nets. * Learners can use the nets they drew in 8Gs7 to calculate the surface area and explore if different nets will render different areas. | Objects to be packaged, paper and scissors |
| 8Gs8 | Identify all the symmetries of 2D shapes. | * Work by families, i.e. all triangles, then quadrilaterals, etc.Recap learners’ understanding of symmetries of 2D shapes. Start with triangles, then quadrilaterals. Use paper shapes to remind learners about line symmetry and rotational symmetry. *What can you say about a shape with 1 line of symmetry?* (e.g. there is only one way that you can fold the shape so that the two parts match) … *order of rotational symmetry 1?* (it fits onto its own outline only once; it has no rotational symmetry)      * Learners work in pairs. Challenge them to try to complete this sorting diagram, by sketching a shape of their choice into the grid. Discuss some different ideas as a whole class.  |  |  |  | | --- | --- | --- | |  | No rotational symmetry | Rotational symmetry | | 1 or 2 lines of symmetry |  |  | | No lines of symmetry |  |  | | More than 2 lines of symmetry |  |  |  * Give learners some properties of a shape, e.g. 1 line of symmetry, four sides, no rotational symmetry. Ask them to sketch the possibilities. (Kite.) * Learners work in pairs, giving similar instructions to their partner, challenging them to draw the shape. | Large paper 2D shapes to demonstrate line symmetry and rotational symmetry  Mirrors and tracing paper as required | |
| 8Ma1 | Know the definition of a circle and the names of its parts; know and use formulae for the circumference and area of a circle. | * *What is a circle?* (A 2D shape whose boundary consists of points equidistant from a fixed point – the centre). You could relate this back to the method of constructing a circle given three points on its circumference. Link type of units to the different formulae for length and area. * Learners make a poster showing the parts of a circle with their definitions, including ‘radius’, ‘diameter’, ‘circumference’. * Learners estimate the length of the circumference of a range of circles given the diameter. They then measure the circumference, using string and a ruler, and tabulate the results. *What pattern do you notice?* (The circumference is just over 3 times the diameter.) * Learners draw round their circles on centimetre squared paper. They count the squares and partial squares to find the area of the circles. * Introduce the concept of pi and the formula *C* = 2π*r* (or *C* = π*D*). * Learners use the formula *C* = 2π*r* to find the circumference of the circles given the radius or diameter*. How does the calculated circumference compare to the measured one?* * Introduce the formula *A* = π*r*2. Learners use the formula *A* = π*r*2  to find the area of their circles. *How does the calculated area compare to the counted one?* * Give problems involving the areas created by combining circles, e.g. *What is the shaded area?* *How did you work it out?* Learners should use their understanding of pi to estimate the answers first.   8cm  12cm | Large sheets of paper    Plastic circles, cylinders, tape measures, rulers, string. | |
| 8Gs9 | Use a straight edge and compasses to construct:   * + the midpoint and perpendicular bisector of a line segment   + the bisector of an angle. | * Go over the construction of the midpoint and perpendicular bisector of a line segment. *Why does this construction work?* (You are finding points that are equidistant from each end of the line segment and all points on a perpendicular bisector are equidistant from each end of a line segment.)   Learners practise the process using a range of line segments in different orientations.   * Learners give instructions to a partner to carry out the above construction. (The partner must carry out the instructions exactly!) * Explain that an angle bisector divides an angle into two equal parts. Learners explore possible ways to construct the bisector of an angle using previous experience. When the learners derive the method they share it with the whole class. If learners find this difficult, show an image that they can point to and use to support their instructions:      * Learners practise the construction of angle bisectors for a range of angles. A partner uses a protractor to check the accuracy of each construction. * Learners give instructions to a partner to carry out the above construction. * Challenge learners to use both these constructions (perpendicular bisector of a line and angle bisector) to construct a rhombus. Ask them to construct a range of different rhombi and link the construction to the properties of a rhombus (diagonals bisect at right angles and bisect the angles at the vertices). | Pairs of compasses  Straight edges (e.g. rulers)  Protractor | |
| 8Gp2 | Transform 2D shapes by rotation, reflection and translation, and simple combinations of these transformations. | * In pairs, learners recap all that they know about transformations from Stage 7. They should take it in turns to talk about: reflection in a given line, rotation about a given point and translation. * In small groups, learners create a poster for one of the transformations above. They present the poster to the rest of the class. There should be posters which cover reflection, rotation and translation.Learners find examples of reflections, rotations and transformations in patterns around the classroom/school. They describe these patterns using the language of transformations. Ask: *Where is the centre of rotation? Where is the mirror line? Describe the translation that has formed that pattern.* * Give each pair of learners an example of a shape on a coordinate grid that has undergone a single transformation. *What transformation has taken place? How do you know? How would a reflection/rotation/translation look different? How would it look the same?* * Learners work in pairs with a simple shape on a coordinate grid. They describe a single transformation using either rotation, reflection or translation, and their partner draws the resulting transformation*. What information is needed to describe your transformation accurately? How does the coordinate grid help you to know where the transformed shape will be?* * Learners carry out the activity above using a combination of two simple transformations. *Would the final shape have the same coordinates if the transformations were done in the opposite order? Why?* * Learners investigate how repeated reflections can be used to generate a tessellation of rectangles. | Large sheets of paper  Squared paper  Coloured pens  Rulers  Pre-prepared transformations on coordinate grids | |

Problem Solving

Below are some possible activities linking the problem-solving strand to the knowledge and understanding learning objectives for the unit. To enable effective development of problem-solving skills, you should aim to include problem-solving opportunities in as many lessons as possible across the unit.

| Activity ideas | Resources |
| --- | --- |
| Draw the line *y* = *x* on the set of axes numbered from -10 to +10. In pairs, learners discuss how they could describe this line so that someone else could draw it on a set of axes*.*  Repeat with the line *y* = 3*x*. |  |
| Learners construct tables, mappings and graphs for *y* = 4*x*, *y* = 1/2*x* and 2*y* = *x*.  *How can you describe the coordinates of any point on the line? What are the equations of the lines? Why are the last two graphs the same?* |  |
| Learners investigate how many different expressions they can create such that they can make a given answer when given values of *a* and *b* are substituted, e.g.  Make expressions that give the answer 20 when *a* = 2 and *b* = 4. |  |
| Learners create and solve 'Think of a number' problems, such as:  I think of a number. I double it and add 4. I divide the answer by 7. I multiply the answer by 2. The result is 4.  What was the starting number? Why? |  |
| A 4 × 2 rectangle can be cut into squares along its grid line 3 different ways.  Ask learners: *How many different ways can a 6 × 3 rectangle be cut into squares?*  *Can a 5 × 3 rectangle be cut into 7 squares? 8 squares? 9 squares? Give mathematical reasons for your answers.* |  |
| This patchwork tablecloth made from 25 squares has 2 lines of symmetry and rotational symmetry of order 2. Learners design other tablecloths of different sizes with the same symmetries.  *Can you find a way of working out how many colours would be needed for an* n *by* n *tablecloth (where* n *is odd)?*  Learners design other tablecloths with different symmetries, explaining how many colours would be needed for different size cloths. |  |
| An old castle has a round table in one room. The diameter of the table is 5.5 metres. A book about the castle claims that 50 people sat around the table. Is the claim true? Justify your reasoning. |  |
| Design a running track to meet these constraints:  The inside perimeter of the track has this shape    Both straights must be at least 80m  Both ends must be identical semicircles  The total inside perimeter must be 400m.  What is the greatest area the running track can enclose? |  |

Unit 2C: Handling Data, Measure and Problem Solving

Data Handling and Measure

| **Curriculum framework codes** | **Learning objectives** | **Activity ideas** | | **Resources** |
| --- | --- | --- | --- | --- |
| 8Ml2 | Know that distances in the USA, the UK and some other countries are measured in miles, and that one kilometre is about 5/8 of a mile. | * Explain that in some countries the unit ‘miles’ is used instead of ‘kilometres’. Tell learners what the relationship between miles and kilometres is, then ask them to complete this table:  |  |  | | --- | --- | | Kilometres | Miles | | 8 | 5 | | 10 |  | | 20 |  | |  | 26.2 (a marathon) | | 100 |  | |  | 100 |   *How can you work out how many miles … kilometres is? How can you check?* (converting in the other direction)   * Learners solve problems such as: Chuck drives 245 miles for his holiday. Amil travels 350 km by bus. Who travels the farthest? * In pairs, learners give each other a distance in one unit of measure, and the partner has to convert to the other unit. | |  |
| 8Ma2 | Derive and use formulae for the area of a triangle, parallelogram and trapezium; calculate areas of compound 2D shapes, and lengths, surface areas and volumes of cuboids. | * Learners work in small groups to derive the formulae from the formula for the area of a rectangle by drawing a range of parallelograms on squared paper. Learners then move a right-angled triangle to form a rectangle.     *What is the formula for the area of a rectangle? So what is the formula for the area of a parallelogram?* Learners derive the formula:  Area of parallelogram = base × height.   * Learners draw parallelograms and calculate their areas and also draw parallelograms of given areas. * Learners work in small groups and draw a range of pairs of congruent triangles on squared paper. They join their triangles to make a series of parallelograms.     Image result for area of triangle  *What is the formula for the area of a parallelogram? What fraction of the area of the parallelogram is the area of one of the triangles? So what is the formula for the area of a triangle?*  Learners derive the formula:  Area of a triangle = ½ (base × height)   * Learners draw triangles and calculate their areas. (They could use their drawings from the previous activity.) * Learners draw a trapezium on squared paper. They then make two copies of this trapezium on card and cut them out. They label one parallel side of each trapezium *b*1 and the other parallel side *b*2. By flipping one trapezium, they join the two trapezia together to make a large parallelogram.   *What is the formula for the area of the parallelogram? So what is the formula for the area of one of the trapezia?*    b1 b2  h  h  b2 b1   * Learners work in small groups. They each draw a trapezium and then calculate the areas of their own and the other learners' trapezia. (Learners could instead use the trapezia created for the activity above.) * Learners use tangram pieces to create different compound shapes (without using all of the pieces). They calculate the area of each image. *How can you check your area?* (The sum of their areas and the areas of the remaining pieces should match the area of the tangram.)      * Learners work in small groups with cuboid-shaped boxes. They discuss, using their previous knowledge from Stage 7, how to find the volume and surface area of a cuboid by measuring lengths and calculating. *What is the formula for the volume … surface area? Why?*   Invite a learner to use a cuboid net to model the derivation of the surface area to the class:  Total surface area = sum of surface area of all 6 faces  = (2 × width × length) + (2 × width × height) + (2 × length x height)  = 2 (*wl* + *wh* + *lh*) | | Basic plastic shapes that can be used to build up compound shapes.  Tangram pieces  Cuboid-shaped boxes  Rulers  Cuboid net  A variety of different nets  Nets for a wide range of 3D shapes  Objects to package  Card  Scissors  Rulers |
| 8Dp2 | Draw, and interpret:   * frequency diagrams for discrete and continuous data * pie charts * simple line graphs for time series * stem-and-leaf diagrams. | * Give learners the raw scores for a class maths test. They use them to construct a frequency table showing the number of learners who achieved each grade.   *How can we represent this data as a pie chart?* Remind learners of the steps for drawing the first sector of the pie chart:   * Work out the total number of learners. * Work out the fraction of the total that got the highest grade in the frequency table. * To work out the angle, multiply the fraction by 360 (because there are 360° in a full turn). * Draw a circle and a radius and measure the angle to draw the sector.   Learners continue to construct their own pie chart. *What mustn't you forget when you draw a pie chart?* (e.g. A title and a key.) Repeat the activity with some continuous data.   * Give learners two pie charts, representing the test grades for a maths test. One pie chart should represent the grades of a single Grade 8 class and the other pie chart should show the results for several Grade 8 classes.   In groups, learners discuss what the individual pie charts show and make comparisons between them.  Take feedback. Encourage learners to use percentage estimates when describing their conclusions.  *Why are pie charts a good way of representing this data?* Establish that pie charts work best for categorical data. They can be helpful when comparing different sample sizes because proportions can be compared visually.   * Explain that you are going to introduce a new type of diagram to display a set of data.   Use the example of test data. Explain that a test is marked out of 50. Display the test marks:  7, 36, 41, 39, 27, 21, 24, 17, 24, 31, 17, 13, 31, 19, 8, 10, 14, 45, 49, 50, 45, 32, 25, 17, 46, 36, 23, 18, 12, 5.  Then show the image of the equivalent stem-and-leaf diagram:  **The key shows us how to read the diagram**  **5 means 25**  **KEY: 2**  **5 is recorded as 05**    **This number is 39**    In pairs, learners discuss how this diagram is formed from the data. Take feedback. *Why is a key important?* After that, provide learners with some stem-and-leaf diagrams for them to interpret.   * Show learners a line graph comparing the average monthly temperature in two places.     Learners discuss the graph in pairs and write down three facts from the graph.  *What features does a line graph always need?* (e.g. title, axis labels.) *What do the shapes of the graphs tell you? How is the temperature similar … different in the two places*?   * Learners use data for monthly temperatures for two locations in the world. Learners construct a line graph to show and compare the average monthly temperatures. | | Raw data of test scores for maths class  Pairs of compasses  Rulers or straight edges  Prepared pie charts showing test data from a number of classes  Stem-and-leaf diagram  Line graph  Data for monthly temperatures |
| 8Di2 | Compare two distributions, using the range and one or more of the mode, median and mean. | * Learners work in small groups. Give learners mark data for a mathematics test and a science test, each displayed as a stem-and-leaf diagram. They calculate and record the range, mean, median and mode. *Can you also find the modal class?* * Ask learners to describe what each statistic means in the context of the mark data (e.g. The range is the difference between the highest and lowest marks.) * *Which of the statistics are helpful in comparing the two sets of data? Which are less helpful?* (e.g. The mode tells you about a few learners' marks and nothing about the others.) * Ask learners to imagine that a teacher needs to present the test data to the principal so he/she can compare the results of different classes. *How might the teacher present the data?* Learners use their chosen method to present the data, justifying their choice. | | Stem-and-leaf diagram |
| 8Di3 | Compare proportions in two pie charts that represent different totals. | * Give learners two pie charts showing some information about the ages of people in France and Greece.   France (12 million) Greece (6.5 million)  Over 59  40–59  Under 15  15–39  Over 59  40–59  Under 15  15–39  Learners discuss questions such as *Roughly* *what percentage of people in France are aged 40–59?* Take feedback. Encourage learners to use percentage estimates when describing their conclusions.  Bashir says ‘The charts show that there are more people under 15 in Greece than in France’. Learners discuss in pairs whether he is right or wrong, and why.  Learners make similar statements for their partner to answer.  *Why are pie charts a good way of representing this data?* Establish that pie charts work best for categorical data. They can be helpful when comparing different sample sizes because proportions can be compared visually.   * Learners collect and use data on travelling to school, and draw two pie charts, one for their class and one for the whole school. They compare and discuss the similarities and differences. | Prepared pie charts  Data showing how learners journey to school by class | | |

Problem Solving

Below are some possible activities linking the problem-solving strand to the knowledge and understanding learning objectives for the unit. To enable effective development of problem-solving skills, you should aim to include problem-solving opportunities in as many lessons as possible across the unit.

| Activity ideas | Resources |
| --- | --- |
| Learners sketch a range of cuboid boxes that hold exactly 16 centimetre cubes. *Which box has the largest / smallest surface area? Convince your partner.* |  |
| 12 cubes with 1 cm edge are each covered in sticky paper. *How much paper is needed?* The 12 cubes are wrapped in a single parcel. *What arrangement of the cubes would need the least paper?* |  |
| A net of a tray is made from a piece of card 18 cm by 18 cm as shown.    Amil says that ‘when the area of the base is equal to the area of the four sides, the volume of the tray will be at maximum.’  Samir does not agree. *Who is correct? Would the result be the same for different size squares?* |  |
| Learners design a helicopter using the diagram, and add a paper clip to the bottom for weight.  Learners drop the helicopter from a set height and time how long it takes to land.  cut  folddd  *What happens if you alter the wing size? The weight?*  Learners draw a graph of their results.  *What type of graph will be the best to use?*  Learners design their own helicopters for optimum flight. | Helicopter diagram, paper clips. |
| Learners investigate data, such as monthly rainfall in two countries over a period of a year. Learners draw a line graph for each country and state a number of facts about it, such as: trends, highest rainfall, drier country, similarities and differences between the countries. | Rainfall data |
| The cost of exporting 350 wide-screen televisions is made up of: Freight charges $371, insurance $49, packing $280 and port fees which are 1/6 of the total. Find the mean cost of exporting a TV.  Construct a pie chart showing how the above export costs make up the total cost. |  |

Unit 3A: Number, Calculation and Problem Solving

Number and Calculation

| **Curriculum framework codes** | **Learning objectives** | **Activity ideas** | **Resources** |
| --- | --- | --- | --- |
| 8Ni3 | Calculate squares, positive and negative square roots, cubes and cube roots; use the notation √49 and 3√64 and index notation for positive integer powers. | * Ask learners to write down as many square numbers as they can remember in one minute. * List square numbers on the board using the notation: 12 = 1, 22 = 4, 32 = 9, 42 = 16 …  102 = 100. *What patterns do you notice?* (e.g. An odd number squared is odd.)   Introduce (-2)2 = 4. Ask learners if this is true or false and ask for explanations. Ask the learners if you can ever get a number that is a square and negative.   * Ask learners square roots of common squares, then ask: *do you know the square root of 30?* Learners discuss what the value could be to develop an understanding that √30 lies between 5 and 6. * Recap positive integer powers of 10 by writing the following list on the board:     103 = 1 000  104 = 10 000  105 = 100 000  106 = 1 000 000  *What patterns do you notice?* (e.g. 102 has two zeros, 103 has three zeros …). *What do you think 10100 would be?*   * In small groups, learners generate similar tables for positive integer powers of 2, 3, 4, 5 and 6. Learners may need reminding that the power refers to the number of times a number is multiplied by itself and this does not equate to the number of zeros as for 10x, e.g. 34 = 81 not 30 000. *What is (-3)2 … (-3)3 … (-3)4? Why?* (The product of two negatives is positive; the product of a negative and a positive is negative.) *What pattern do you notice?* * In pairs, learners make a set of 12 dominoes using squares and square roots; cubes and cube roots. (The dominoes should create a closed loop.) * Learners work in small groups to construct a range of larger cubes out of interlocking cubes from a 2 × 2 × 2 cube up to a 5 × 5 × 5 cube. They list the number of cubes that they needed to construct each cube.   Display the results introducing the vocabulary ‘cubed’ (to mean ‘to the power of 3’) and ‘cube root’ and the notation for cube root (e.g. ³√64), e.g.  13 = 1 ³√1 = 1  23 = 8³√8 = 2 | Scientific calculators  Card and scissors  Interlocking cubes |
| 8Nf7 | Simplify ratios, including those expressed in different units; divide a quantity into more than two parts in a given ratio. | * Recap simple ratios using coloured cubes. Ask learners to create the following ratios using any number of cubes they like:   2:1  3:1  3:2  After each arrangement, learners feed back to the whole group explaining different solutions.  Explain how ratios are simplified, for example, if learners have illustrated 3:1 as 9 blue cubes:3 red cubes they have simplified 9:3 to 3:1.   * Develop the first activity above to create a three-part ratio, for example 3:2:1. Learners work in pairs and feed back on their solution. *What strategy did you use?* *What other solutions are there?* Learners find other examples of dividing the cubes into three parts. * Learners work in pairs to use cubes to create a four-part ratio. They identify the ratio they have created. *What strategy did you use?* *What other solutions are there?* * Using cubes, model a ratio problem such as:   3 children aged 3, 4 and 5 are given $60 by their mother. They share the money in the same ratio as their ages.  How many dollars does each child receive?  Begin by dividing the cubes into 3 piles, putting 3 cubes in the first pile, 4 in the second, 5 in the third and then repeating.  *Can you see a quicker way of calculating the answer?* Model one round of the sharing again. *How many cubes have I shared out so far? So how many rounds of sharing can I do with all 60 cubes? So how many cubes does each pile get?*   * Give learners some ratios to simplify, such as 2.5:25; 20mm:4cm; 48:64:128; 1/2:75%:0.25. * Give ratio problems like the one above for learners to solve in pairs (using diagrams for support if needed). Include problems that include different units, e.g.   A recipe always uses eggs, flour and butter in the same ratio. If I need 2 eggs, 50 g of butter and 0.5 kg of flour for one cake how much of each ingredient do I need for three cakes?  *How did you work out your answer? How can you check your answer?*   * Learners create ratio problems for a partner to solve (they must be able to calculate the answer themselves). *Explain the strategy that you are using to make up your problems … solve your partner’s problems.* | Coloured cubes (e.g. interlocking cubes) |
| 8Nf8 | Use the unitary method to solve simple problems involving ratio and direct proportion. | * Recap the kilometre-to-mile conversion that was introduced earlier. Ask learners to complete the following table:  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | | km | 8 |  |  |  |  | | miles | 5 | 10 | 20 | 25 | 50 |   Model the unitary method. Establish that if 8 km = 5 miles then 1 km = 5/8 mile. Explain that using the unitary method (working from 1 unit) allows you to calculate any conversion very quickly. For example, to find 32 km in miles we calculate:  32 × 5/8 or 32 × 0.625 = 20 miles  Show this is correct by referring to the table.   * Give learners a range of problems focusing on direct proportion in real-life contexts, for example: * A company can make 5 specialist cars in 2 weeks. How many can they produce in a year if they work at the same rate every day of the year? * An author can write 750 words in 1 hour. They write for 7 hours a day. How many words do they write in 1 week if they work every day of the week?   *What information is given in the question? What extra knowledge do you need to use?* (e.g. There are 52 weeks in a year.)  Choose learners to present their strategies and solutions to the rest of the class. Include the unitary method in your discussions.   * Learners create word problems focusing on direct proportion for a partner to solve. (They must be able to solve them themselves.) *How did you decide what information to give in your problem?*   They compare and discuss strategies. |  |
| 8Nc4 | Use known facts and place value to multiply and divide simple fractions. | * Give examples of simple fraction multiplications, such as   ½ × ¼ ¾ × ½  Ask some learners to provide the answers orally and then explain their strategy.  Establish that the multiplication sign is equivalent to ‘of’ so  ½ × ¼ is equivalent to ½ of ¼  ¾ × ½ = ½ × ¾ which is equivalent to ½ of ¾  Learners draw diagrams to illustrate the answers.  e.g.  Record the answers to the calculations:  ½ × ¼ = 1/8  ¾ × ½ = 3/8  Then show them the multiplication ½ × ½. Explain that this is half of half and therefore they could use decimals and known facts to solve it. *How much is half of 0.5? (0.25) How can I represent 0.25 with a fraction?*   * Give examples of simple division calculations such as:   ½ ÷ 1/6  ¾ ÷ 1/8  Remind learners that the questions are asking how many of the second fraction is ‘in’ the first one.  Learners draw diagrams to illustrate answers.  Record the answers to the calculations:  ½ ÷ 1/6 = 3  ¾ ÷ 1/8 = 6  *Can you identify a rule for dividing fractions?* Use the fact that multiplication and division are inverses to explain the algorithm. |  |
| 8Nc9 | Use the laws of arithmetic and inverse operations to simplify calculations with integers and fractions. | * Learners work in pairs. They select five digit cards at random and use the digits and two of the operations +, –, × and ÷ to create 5 calculations. Each learner creates and the others says the answer to the calculation. The first learner tries to guess which strategy the second learner used to answer. Then the other can explain the strategies they used to simplify the calculations, e.g. using laws of arithmetic:   (97 + 24) × 5 = (97 + (3 + 21)) x 5  = 121 × 5  = (100 × 5) + (21 × 5)  = 500 + 105  = 605  (97 + 24) × 5 = 121 × 5  = 121 × 10 ÷ 2  = 1210 ÷ 2  = 605  *How can you check your answer?*(Using inverse operations.)   * Learners use the digits 3, 4, 5, 6, 7 and any of the operations +, –, × and ÷ to create and find the answer orally to a calculation. The digits must appear in order, for example 345 × 67 or 3 × 45 + 67. Learners create 10 different calculations. *How can you estimate the answer first? How can you simplify the calculation to help you calculate? How can you check your answer?* * Recap adding and subtracting fractions using examples with mixed numbers. Ask learners to discuss in pairs different ways of calculating, e.g. * 35/8 + 15/12 * 123/4 – 75/6. * Discuss different mental strategies as a class. Include: * converting to improper fractions before converting to fractions with common denominators, e.g.   35/8 + 15/12 = 29/8 + 17/12 …   * using laws of arithmetic, e.g.   123/4 – 75/6 = (12 + 9/12) – (7 + 10/12)  = 11 + 19/12 – 7 – 10/12  = (11 – 7) + (19/12 – 10/12 )….   * using inverse operations, e.g. counting on from 75/6 to 123/4:   + 1/6 (to get to 8)  + 4 (to get to 12)  + 3/4 (to get to123/4)  So 123/4 – 75/6 = 1/6 + 4 + 3/4. | A set of 0–9 digit cards for each pair  A set of operation cards (+ - x and ÷) |
| 8Nc10 | Use the order of operations, including brackets, with more complex calculations. | * Recap the order of operations and the use of brackets. Write a simple calculation, for example 3 + 4 × 6 – 7, on the board. Ask learners to imagine that there is no standard order of operations. *How many different answers would we be able to make?* Learners come to the board and add brackets to change the result of the calculation. *What is the correct answer if we use the standard order of operations? Why?* * Learners work in pairs and use each of the numbers 1, 2, 3, 4 and the operations +, –, × and ÷ to try to make every number from 1 to 20 without writing it down. They need to tell the partner their rationale, e.g.   1 = 4 – 3  2 – 1  They can include indices and roots.   * Challenge learners to place the operations +, –, × and ÷ between the digits 1, 2, 3, 4, 5, 6, 7, 8, 9 to make a calculation with the answer 100, e.g.   (1 + 2 + 3 + 4) × 5 + 67 − (8 + 9) = 100.  *How many different ways can you find?* |  |
| 8Nc13 | Multiply and divide integers and decimals by decimals such as 0.6 or 0.06, understanding where to place the decimal point by considering equivalent calculations, e.g. 4.37 × 0.3 = (4.37 × 3) ÷ 10, 92.4 ÷ 0.06 = (92.4 × 100) ÷ 6. | * Learners work in pairs or small groups to develop and share successful methods for problems.They use the digits 3, 4, 5 and 6 to create as many different multiplication calculations as they can. Each calculation must include a number with 1 decimal place, for example:   3 × 45.6 = 136.8  0.6 × 543 = 325.8  *How did you work out this answer?*  *What is the largest possible product? What is the smallest possible product? How do you know?*   * Learners use the digits 0, 3, 4, 5 and 6 to create as many different division calculations as they can. Each calculation must include the divisor 0.3, 0.4, 0.5 or 0.6, for example:   465 ÷ 0.3  354 ÷ 0.6  *How can you make the answers easier to calculate?* (e.g. divide by 3 first and then multiply by 10.) |  |

Problem Solving

Below are some possible activities linking the problem-solving strand to the knowledge and understanding learning objectives for the unit. To enable effective development of problem-solving skills, you should aim to include problem-solving opportunities in as many lessons as possible across the unit.

| Activity ideas | Resources |
| --- | --- |
| Take a two-digit number. Reverse the digits.  Is it possible to make a number that is one-and-a-half times as big as the original number?  Justify your answer.  Find a two-digit number that is one-and-three-quarters times as big when you reverse its digits. |  |
| Calculate these without a calculator:  4², 4³, 44, 45, 46  What digit does 420 end in?  What digit does 421 end in? Justify your answers. What can you say about powers of 3? |  |
| If the sum of two numbers is 17, what is the greatest product they can have?  Investigate what happens if there are three numbers. |  |
| The distance to Andover is given as 5 miles.  Andover 5 Stoke 2 ¼  Enham 3 ½ Wyke ½  Do you think the distance to Andover is correct to the nearest half mile, the nearest quarter mile or the nearest mile?  Give reasons for your answer. Use your answer to give:   * The shortest and longest distances to Andover * The shortest possible distance between Stoke and Enham. |  |

Unit 3B: Algebra, Geometry, Measure and Problem Solving

Algebra, Geometry and Measure

|  |  |  |  |
| --- | --- | --- | --- |
| **Curriculum framework codes** | **Learning objectives** | **Activity ideas** | **Resources** |
| 8Ae7 | Construct and solve linear equations with integer coefficients (unknown on either or both sides, without or with brackets). | * Learners explore finding three consecutive numbers with a given sum, e.g. ‘Which three consecutive numbers have a sum of 24?’   *How can you write an expression for the sum of three consecutive numbers? How can you use this to find three consecutive numbers with a given total?*  Establish that three consecutive numbers can be generalised as:   |  |  |  | | --- | --- | --- | | *n* | *n* + 1 | *n* + 2 |   So you can find three consecutive numbers with a sum of 24 by constructing and solving:  *n* + (*n* + 1) + (*n* + 2) = 24  3*n* + 3 = 24  3*n* = 21  *n* = 7  So the consecutive numbers are 7, 8 and 9. This activity can be repeated for four or five consecutive numbers.   * Learners investigate sums of four cells in a 100 square,   e.g. *Is there a 2 x 2 square of numbers that have a total of 150?*   |  |  | | --- | --- | | *n* | *n* + 1 | | *n* + 10 | *n* + 11 |   *n* + (*n* + 1) + (*n* + 10) + (*n* + 11) = 150  4*n* + 22 = 150  4*n* = 128  *n* = 32  So the numbers in the 2 x 2 square are, 32, 33, 42 and 43.  Learners then explore shapes that can be made from 5 adjoining cells. *Which arrangements lead to simple expressions? e.g.*  n  n + 1  n – 1  n – 11  n + 11   * Learners construct and solve simple linear equations in the context of perimeters of regular polygons, for example: * A square has perimeter 44 cm. What is the length of its sides?   (4*a* = 44 so *a* = 11 cm)   * A regular hexagon has perimeter 72 cm. What is the length of its sides? (6*b* = 72, *b* = 12 cm).   *What is the formula to find the perimeter of a square … hexagon? How does this help you to form an equation? How can you solve the equation?* (Dividing both sides by …)  Learners then work in pairs to construct and solve more complex linear equations in the context of perimeter, e.g.   * A rectangle has perimeter 48 cm. It is twice as long as it is wide. What is the width of the rectangle? * A rectangle has a perimeter of 16 cm. The width is 2 cm less than the length. How long are the longestsides?   Encourage learners to draw and label diagrams to help them to construct the equations. *How can you solve the equation?*   * In pairs, learners apply their understanding of solving equations to equations with the unknown on both sides, e.g.   2*x* + 2 = *x* + 4  3(5*x* – 4) = 2(2*x* + 5)  They discuss the process and then feed back to the whole class.   * Learners create equations with the unknown on both sides for a partner to solve. They should ensure that the solution is an integer. *What strategies are you using to make sure the solution is an integer? How can you solve the equation?* |  |
| 8As1  8As2 | Generate terms of a linear sequence using term-to-term and position-to-term rules; find term-to-term and position-to-term rules of sequences, including spatial patterns.  Use a linear expression to describe the *n*th term of a simple arithmetic sequence, justifying its form by referring to the activity or practical context from which it was generated. | * Learners count forwards and backwards using sequences such as: * first term 5, term-to-term rule add 12 * first term 52, term-to-term rule subtract 7. * Give out cards showing term-to-term rules for creating a linear sequence, e.g. + 5; – 3; × 3; ÷ 2. Learners create sequences using 1 as the first term. *How can you find the 10th … 20th … 50th …* n*th term? How do you know that rule is correct?* * Learners carry out the activity above but using a die to generate the first term. * Learners write the first 5 terms of their own linear sequence created by adding or subtracting a constant, e.g. 100, 93, 86, 79, 72 ….   They share their sequence with a partner who looks at the sequence and finds the term-to-term rule. *What is the* n*th term rule? How can you describe that using an expression? How do you know your expression is correct? How can you test it?*   * Learners work in pairs to create the sequence of multiples of 2 in a spreadsheet. They create a table showing the position of the term (from 1 to 10) in the left column and corresponding term (starting with 2) in the right column. The first three rows would be:  |  |  | | --- | --- | | **Position** | **Term** | | 1 (A2) | =A2\*2 | | =A2+1 | =A3\*2 | | =A3+1 | =A4\*2 |  * Learners explore and record the effect of adding or subtracting a constant to each term of the sequence of multiples of 2. *What stays the same?* *What changes? What is the* n*th term?* (2*n* + *b*) * Learners create spatial patterns that fit a given sequence, e.g.   e.g. 2*n* + 1   * Learners use a visual representation to generate a linear sequence and investigate the *n*th term rule. * Learners create their own growing stick patterns and find the term-to-term and *n*th term rules. Encourage learners to use their diagrams to explain their reasoning. *How can you check that your* n*th term rule is correct? How did you calculate the* n*th term rule?*   Examples of possible patterns are: | Counting stick or any other material to help learners count forwards and backwards  Prepared sets of cards showing term-to term rules for creating linear sequences  Dice  Spreadsheet software  Visual representations of a linear sequences  Matchsticks |
| 8Gs5 | Understand a proof that   * the angle sum of a triangle is 180° and that of a quadrilateral is 360°. * the exterior angle of a triangle is equal to the sum of the two interior opposite angles. | * Learners use their knowledge of alternate angles to explain why the angle sum of a triangle is 180°. They should show understanding that it is true in all cases.     *d* + *c* + *e* = 180°(angles on a straight line = 180°)  *b* = *d*, *a* = *e* (alternate angles)  therefore *a* + *b* + *c* = 180°.   * In pairs, learners use the above diagram to set problems for each other. They give the values of two angles (135°and 70° as shown) and their partner has to find the values of all the other angles, confirming that the angle sum of a triangle is 180°. * Learners use their knowledge of the angle sum of a triangle to show that the angle sum of a quadrilateral is 360°.   They draw diagrams of two triangles showing that (*a* + *b* + *c*) + (*d* + *e* + *f*) = 180°+ 180°.    c  a b  d e  f   * Learners show that the exterior angle of a triangle is equal to the sum of the two interior opposite angles by completing the following diagram with the respective angles.   Learners explain their reasoning in logical steps showing how one step leads to another. |  |
| 8Gs6 | Solve geometrical problems using properties of angles, of parallel and intersecting lines, and of triangles and special quadrilaterals, explaining reasoning with diagrams and text. | * *What are parallel lines? What is a transversal?* In pairs, learners create a poster to define vertically opposite angles, alternate angles and corresponding angles and illustrate them using a transversal of parallel lines. * Give a range of geometrical problems using: * vertically opposite, corresponding and alternate angles * angles in triangles * angles in quadrilaterals   Learners find missing angles and explain their reasoning. For example:  http://www.mathplanet.com/media/39194/line08.png  C:\Documents and Settings\Home\Desktop\s6q6.gif  http://www.mathsteacher.com.au/year8/ch09_geometry/07_quad/Image15242.gif   * Learners use their knowledge of angle properties of parallel lines, triangles and quadrilaterals to set questions for other learners (they must be able to solve the problems themselves and explain their reasoning). |  |

Problem Solving

Below are some possible activities linking the problem-solving strand to the knowledge and understanding learning objectives for the unit. To enable effective development of problem-solving skills, you should aim to include problem-solving opportunities in as many lessons as possible across the unit.

| Activity ideas | Resources |
| --- | --- |
| In this pyramid, each number is the sum of the two numbers immediately above it. Learners find a value of *n* which makes the bottom number correct.  21  14  *n*  53  Learners find the missing numbers in this pyramid, where each number is the sum of the two numbers below it. Learners find a value for *n* which makes the top number correct.  44  3  *n*  8  2 |  |
| A triangle has sides of *a* + 4, 4*a* – *b*, and *b* + 2. The triangle is equilateral*.* Learners find its perimeter in terms of *a*. |  |
| Learners construct and solve simple linear equations in the context of perimeters of compound shapes and irregular polygons, for example:  Find an equation for the perimeter of this shape. What are three possible values for the perimeter?    (3*x* + 4) cm  9 cm    5 cm    (5*x* + 2) cm  *What extra information do you need to find? How will you do this? Can* x *be a negative integer?* Learners create problems like the one above for a partner to solve.  *Have you given enough information to solve the problem? Have you given more than enough?* |  |
| Give problems involving constructing equations with the unknown on both sides, such as:  *When I am twice as old as I am now I will be three times as old as I was 3 years ago. How old am I?* |  |
| Here are five expressions: 2*x* + 3*y* – 20 5*x* – 2*y* + 38 4*x* + 5*y* – 72  *x* – 4*y* + 108 3*x* – *y* + 39  Learners find the particular value of *x*, and a value of *y* to go with it, which make all five expressions equal in value.  *Did you have more information than you needed? Not enough information?*  *Or exactly the amount required to solve the problem?* |  |
| Learners explore how many squares (of all sizes) there are on a chess board. This leads to a sequence involving square numbers (One 8 × 8 square, four 7 × 7 squares, nine 6 × 6 squares …) *What is the simplest case?* (the whole 8 × 8 square) *What is the rule for moving from case to case? How many squares would there be on a 10* × *10 ‘chess board’?* |  |

Unit 3C: Handling Data, Geometry, Measure and Problem Solving

Handling Data, Geometry and Measure

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| **Curriculum framework codes** | **Learning objectives** | **Activity ideas** | **Resources** |
| 8Gs10 | Use a ruler and compasses to construct   * circles and arcs * a triangle, given three sides (SSS) * a triangle, given a right angle, hypotenuse and one side (RHS). | * Review previous construction skills with ruler and compasses. * Ensure that learners understand the terms ‘radius’ and ‘diameter’. Learners practise drawing circles with a given radius/diameter. *How are you making sure your circle has the correct radius/diameter?*   A partner uses a ruler to check their accuracy.   * Learners create patterns with intersecting circles and colour them. * Model the construction to draw a circle, given 3 points A, B and C, which lie on its circumference (join up the points to form two lines; construct the perpendicular bisector of one line; do the same for the other line; where they cross is the centre of the circle).   B  C  O  A  Learners use their knowledge of perpendicular bisectors to complete the construction.  *Why does this construction work?* (You are finding a point that is equidistant from all three points, i.e. the centre of the circle.)   * Model the construction to draw a triangle, given all three side lengths. Learners practise this construction. * In pairs, learners explore possible ways to construct a right-angled triangle, given the length of the hypotenuse and one side, using their previous experience. If learners derive the method, they share it with the class. If required, model the construction to the class. Learners can then practise the construction. * Learners set construction tasks for a partner to undertake. They should involve drawing a circle through 3 points, drawing a triangle given 3 sides or drawing a triangle given a right angle, the hypotenuse and one side. Learners check the accuracy of each other’s constructions. | Rulers, pairs of compasses, plain paper |
| 8Gp3 | Understand and use the language and notation associated with enlargement; enlarge 2D shapes, given a centre of enlargement and a positive integer scale factor. | * On a large coordinate grid, model the process for enlarging shapes using a given centre of enlargement and a positive integer scale factor. Reinforce that when carrying out an enlargement you need to know the scale factor and the centre of enlargement. The 'scale factor' tells us how much the shape has been enlarged. The 'centre of enlargement' tells us where the enlargement has been measured from.     In this example the centre of enlargement is (2, 2) and the scale factor is 2. To enlarge the shape you measure from the centre of enlargement to any point on the original shape. You then multiply by the scale factor to give you the distance from the centre of enlargement to the equivalent point on the enlarged shape.   * In small groups, learners carry out given enlargements of simple regular shapes on large sheets of squared paper. Give learners the coordinates of the required starting shape. Initially use the origin as the centre of enlargement and scale factor 2, 3 or 4. *How is an enlargement different from other transformations?*   *How can you check your enlargement?*   * Learners enlarge a range of different compound shapes with a given centre of enlargement and scale factor. * In small groups, learners explore the impact of enlarging shapes with a given centre of enlargement (but different scale factors) on the perimeter and area of shapes. They use either equilateral triangles, isosceles triangles, squares or rectangles.   Learners report their findings to the rest of the class. As a class, agree on a generalisation from the investigation. | Large coordinate grid  Large sheets of squared paper. |
| 8Gp4 | Interpret and make simple scale drawings. | * Use maps of the local area to discuss the nature of scale drawings. *What do we mean by ‘scale’? What is the scale of this map? What does that mean? How big would … be in real life?*   *When do you use scale drawings? What other occupations use scale drawings?* If you have access to architectural plans for the school these are excellent examples to share with learners.   * Learners make a scale drawing of the classroom. Make the measurements as a whole class. Discuss what the scale factor should be and model calculating a scaled length.   Learners make their drawings and add scale plans of the classroom equipment to design their ideal classroom.  Learners present their ideas to a small group. Peers give feedback highlighting two strengths and one area for improvement. | Blocks/model building to draw  Tape measures, rulers, squared paper |
| 8Mt1 | Draw and interpret graphs in real life contexts involving more than one component, e.g. travel graphs with more than one person. | * Display a distance–time graph for a journey of two people starting from the same town at different times/speeds. In pairs, learners describe what the graph represents*. How long does A’s journey take? How far does B cycle before meeting A? Who walks faster?*   15  10  5  0  A  B  distance (km)  09:00 10:00 11:00  Time (hours)   * In pairs, learners draw a graph to represent two different people’s journeys.   Select learners to show and describe their new journeys.  *What is the same … different about the two journeys?*   * Describe journeys in words for learners to graph, e.g. Sunil leaves home at 8:30 and walks for 30 minutes at 2 km/h. He rests for 15 minutes before returning home. His total journey takes 1 hour. * In pairs, learners draw a distance–time graph that shows both of their journeys to school on the same graph. *What is the same … different about your journeys? How is this represented on your graph?* | Distance–time graph |
| 8Db3 | Find and list systematically all possible mutually exclusive outcomes for single events and for two successive events. | * Learners roll two different dice or spin two spinners and list all the outcomes. They compare their results with colleagues. * Challenge learners to give the probability of each combination of numbers when throwing two dice. *How can you record your results systematically?* e.g. using a grid of all possible outcomes (a ‘sample space diagram’):  |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | --- | |  | 1 | 2 | 3 | 4 | 5 | 6 | | 1 |  |  |  |  |  |  | | 2 |  |  |  |  |  |  | | 3 |  |  |  |  |  |  | | 4 |  |  |  |  |  |  | | 5 |  |  |  |  |  |  | | 6 |  |  |  |  |  |  |   *How can you check your answers?* (By checking that they add to 1.)   * Carry out experiments to sample the number of unknown counters in a bag. * Suggest how many of each type of counter are in the bag, given a known total. * Give learners examples of pairs of independent events, for example: * tossing the number side of a coin and rolling a 5 on a 6-sided die * choosing an even number from a pack of cards and tossing the number side of a coin * choosing a 3 from a pack of cards, replacing it, and then choosing a 10 as the second card * rolling a 4 on a 6-sided die and then rolling a 1 on a second roll of the die.   In groups, learners list all possible outcomes for the above to calculate their probability. *How will you make sure you have all possible outcomes? How will you calculate the probability of each outcome?*   * Groups discuss the probability of a range of events based on dice and draw them on a probability scale. This should include events using two or more dice, e.g. * throwing an even number * throwing a number greater than 4 * throwing a double with two dice. *What are the possible mutually exclusive events?* | Dice, spinners and counters. |
| 8Db4 | Compare estimated experimental probabilities with theoretical probabilities, recognising that:   * when experiments are repeated different outcomes may result * increasing the number of times an experiment is repeated generally leads to better estimates of probability. | * Learners work in small groups (4s). Each learner rolls a die 20 times, recording the results. The group combine their 4 results to create a larger sample set; they discuss how estimated probability changes as more data is added to the set. All groups add their data to create an even larger data set, discussing the changes that occur. A spreadsheet can be used to graph large quantities of data to show convergence towards a limit. * Pairs of learners throw two 1–6 dice and record the sum of the scores. They repeat 25 times and record on a frequency diagram. They compare the results with another pair. *Are they different? Why?* Predict what might happen if the experiment were repeated. Carry out the experiment another 25 times and record the results on the same diagram. *What effect did the extra score have?*   *Did the results match the predictions? Why?*   * Learners play ‘The Horse Race Game’, where 12 numbered horses move along a row of squares towards a finish line. Learners take turns to throw two dice and the horse that represents the total is moved forward one space.   Learners predict which horse will win before they play. Each group plays 3 or 4 times then shares their results with the class. *Are you surprised by your results? Why?*  *How can we explain the likelihood of horse 1 winning? … horse 2 winning? …* They use this to discuss the results of the game (e.g. 1 is impossible. There is only 1 way to throw a 12.)   * The results from an experiment where a drawing pin was dropped 1000 times are 279 times pin up, 721 times pin down.   Explain that the relative frequency of landing pin up is 279/1000 or 29.7%. Learners replicate this experiment, comparing and discussing the results. | Dice  Spreadsheets  Horse race game  Drawing pins |

Problem Solving

Below are some possible activities linking the problem-solving strand to the knowledge and understanding learning objectives for the unit. To enable effective development of problem-solving skills, you should aim to include problem-solving opportunities in as many lessons as possible across the unit.

| Activity ideas | Resources |
| --- | --- |
| Ask learners to enlarge a scalene triangle by a scale factor of 2 about the first centre of enlargement. Now enlarge the resulting triangle by a scale factor of ½ about the second centre of enlargement. *What do you notice? Would it be possible to go from the first triangle to the last triangle with only one enlargement?*  Learners try this with the triangle in different positions, and with different centres of enlargement. They describe what will happen for any two centres of enlargement and a triangle in any position.  Learners try changing the scale factors so that the first enlargement is by scale factor 3 and the second by scale factor . *What do you observe?* |  |
| Fifteen learners measured an angle. Their results are:  Angle measured Number of learners  45° 5  134°3  135° 4  136° 3  Use the results to decide what the angle is most likely to measure. Give your reasons. |  |
| Learners sketch a graph which would show:   * something increasing steadily * something staying the same * something which stays the same and then suddenly decreases * something which increases steadily and then decreases rapidly.   They compare their answers with a colleague and then decide which they want to present to the rest of the class. They need to explain why they chose that one and why it is a good representation of the situation. |  |